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Force of Attraction and Partner Availability in the U.S. Marriage Market: A Two-Sided Matching Model

Yuan Cheng,[a] John K. Dagsvik[b], Xuehui Han[c], Zhiyang Jia[b]

a) Fudan University; b) Statistics Norway; c) International Monetary Fund

Abstract: This article develops and applies a stochastic two-sided matching model to analyze marriage patterns in the United States using 1 percent samples from the 2010 and 2019 American Community Survey, accessed via the Integrated Public Use Microdata Series. This approach disentangles two sources of change in marriage patterns over time: individuals' preferences for partner characteristics ("forces of attraction") and the numbers and composition of potential partners ("partner availability"). As illustrated by our empirical application, the model provides a flexible and unified analytical framework to address a broad range of relevant questions in marriage research, offering valuable new perspectives on marriage dynamics and facilitating future research, despite the limitation that the model does not separately identify individual-specific preferences.

Keywords: stochastic two-sided matching model; marriage; forces of attraction; partner availability

Reproducibility Package: https://github.com/GuoguomaFD/Matching.

THIS article seeks to improve our understanding of a longstanding challenge in the analysis of marriage: to what extent changes in marriage patterns are driven by shifts in individuals' preferences for particular personal characteristics of potential partners (which we refer to as "forces of attraction") and to what extent they stem from changes in "partner availability"–i.e., the numbers of participants with different characteristics, such as age, education, and race, on both sides of markets.

For this purpose, we employ a stochastic two-sided matching model grounded in both random utility theory (McFadden 1984) and stable matching theory (Gale and Shapley 1962). This model assumes that each side of the marriage market composed of a large set of individuals, who are characterized by observed attributes, such as age, education, and race, as well as unobserved attributes. An individual's characteristics not only affect his or her own preferences but also enter as attributes in the utility functions of potential partners. This model can be used to quantitatively estimate joint surplus of matching as a function of both partners' characteristics from data on realized matches. These estimates are key to characterize and simulate matching outcomes under various counterfactual scenarios, allowing us to shed further light on many interesting questions within the marriage market literature.

The two-sided matching framework is well established theoretically and offers a useful complement to the log-linear and conditional logit approaches that dominate empirical studies of marriage markets. This article applies the two-sided model to recent U.S. marriage patterns to showcase the additional insights it provides. Similar to conditional logit models, it addresses key limitations of log-linear

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specifications (Gullickson 2021), and it goes further by explicitly modeling the underlying marriage market and allowing individuals to remain single–features that enable a clearer separation of the forces driving observed marriage outcomes.

Although the number of marriages changes over time, existing methods–such as harmonic mean functions–cannot assess the statistical significance of these shifts. Using our two-sided matching model, we can address this in a tractable way. We introduce a likelihood-ratio test that formally evaluates whether changes in the attraction-parameter estimates are significant. We also propose alternative ways for constructing commonly used inferences, such as the impact of age difference on utility (Logan, Hoff, and Newton 2008), odds ratios assessing age and education homogamy (Qian and Lichter 2018), and the status-merit exchange index (Xie and Dong 2021).

Our empirical analysis primarily draws upon the 1 percent samples from American Community Survey (ACS) for 2010 and 2019, provided by Integrated Public Use Microdata Series (IPUMS). We opt for 2019 over 2020 to circumvent potential distortions attributable to the Covid-19 pandemic. In addition, we refrain from extending our analysis to earlier periods due to limitations in the survey data from 1990 to 2000, which lack sufficient information to ascertain marriage years.

Our empirical results suggest that, on average, there has been a notable decline in the forces of attraction of marriage relative to remaining single over the past few decades. However, this change is not uniform across groups. Although the forces of attraction between middle-aged, college-educated women and men of other races remain the highest in 2019, it is substantially lower than in 2010; in contrast, the forces of attraction between middle-aged, college-educated black men and middle-aged, high-school-educated black women increase from 2010 to 2019.

Assortative forces of attraction, or homogamy, remain strong, though slightly lower in 2019 than in 2010. This persistence of assortative mating is consistent with patterns documented for the 1990s (Jepsen and Jepsen 2002) and for 2008–2014 (Qian and Lichter 2018). These trends suggest that individuals continue to prefer partners with similar characteristics, such as age, education, and race.

We also find that changes in partner availability between 2010 and 2019 have overall increased the number of marriages: Our results suggest that the change in partner availability could have resulted in 7,824 more marriages in 2019 compared to 2010, representing around a 45 percent increase compared to the observed number of marriages in 2010. However, there are exceptions. For example, young white men with high school education experienced a reduction in marriage attributed to changes in partner availability. Young white women in the same education group saw a similar decline, potentially because of increased educational attainment among young individuals.

Finally, our findings provide updated perspectives on the existing literature along several dimensions. First, we document stronger educational and age homogamy than reported by Qian and Lichter (2018), and a level of age homogamy comparable to that documented by Logan et al. (2008). Second, we find strengthening positive status-merit exchange between race and education from 2010 to 2019, offering an alternative perspective to the findings obtained using the approach of Xie and Dong (2021). Third, our analysis suggests that the decline in marriage rates among young black women with a high school education is driven primarily by weakening within-group attraction and, to a lesser extent, by a shrinking pool of potential partners. We also find that, contrary to concerns that higher educational attainment reduces marriage utility, the forces of attraction for college-educated women and men in the middle-aged group remains the highest across racial groups, despite being lower in 2019 than in 2010. Finally, despite sex imbalances arising from international immigration, partner availability appears to have broadened for both men and women, although the expansion is more limited for women.

Although this application provided valuable insights into marriage patterns over time in the United States, it is crucial to interpret these findings in light of the model's underlying assumptions and the specific limitations of our empirical implementation. In the literature on two-sided matching models, it is well known that, with only the marriage outcome data, it is not possible to separately identify men's and women's preferences without imposing further restrictions, even though the joint surplus of matching is identifiable (Dagsvik 2000; Menzel 2015; Dagsvik and Jia 2022). This negative identification result consequently limits the framework's ability to capture gender differences in matching unless extra structure is imposed.

In our article, when knowledge on individual specific preferences is needed, we make a simplifying assumption that husband and wife share the joint surplus equally. An alternative approach, as in Diamond and Agarwal (2017), is to assume that the systematic components of payoffs are homogeneous across individuals and follow a specified parametric functional form. In contrast, our framework allows for non-parametric heterogeneity across different groups. We do not consider the parametric alternative to be clearly superior, as any mis-specified functional form could bias estimates of joint surplus and lead to questionable predictions of matching outcomes. One source of reassurance in our approach is that the model estimates a utility level for each match category, that is, the maximum utility a husband or wife can obtain from the set of potential partners, including the option of remaining unmatched. Because each category corresponds to a specific combination of age, education, and racial differences between spouses, the estimated utilities allow us to assess how utility varies across observable partner characteristics. For example, as simulated using our 2010 estimates (see Section 5.2), match categories in which the wife is approximately two years younger yield the highest utility for men, whereas categories in which the husband is about two years older yield the highest utility for women.

In the empirical analysis, we assumed that individuals operate in a national marriage market. This might have been an oversimplification, as partner selection is often local and the distribution of characteristics varies geographically. However, we note that the evolving landscape of how people meet, particularly the increasing role of online dating platforms, may help mitigate this concern to some extent. Online dating has been shown to transform partner search by potentially expanding individuals' effective choice sets beyond traditional geographic boundaries. A potentially more serious concern is the issue of data sparsity. With 18 types on each side of the market, there can be few observations in some of the cells. This may have contributed to the instability of some estimates over time. In addition, the model incorporates only a limited set of characteristics that influence partner attractiveness. For example, it does not account for history of incarceration, which is particularly important for certain groups, such as young black men with low levels of education. A history of incarceration, even when individuals are no longer incarcerated, may significantly affect their attractiveness in the marriage market. In its current form, the model attributes changes in marriage patterns not explained by shifts in demographic composition to changes in preferences. As a result, it could misleadingly attribute lower attractiveness to young, low-educated black men as a group, when in fact the reduced attractiveness may primarily stem from the subgroup who have been incarcerated. These caveats should be kept in mind when interpreting the empirical results.

Despite these limitations, we believe the stochastic two-sided matching model remains a valuable and flexible tool for analyzing marriage markets, offering distinct advantages over conventional approaches in certain respects. Future research can build on this framework by incorporating more detailed data, refining the specification of the set of market participants, and explicitly modeling additional factors that influence marriage formation.

The subsequent sections of this article are structured as follows: Section 1 discusses the related literature. Section 2 summarizes the theoretical model. Section 3 outlines the data utilized in this study. Section 4 presents and deliberates upon the empirical findings, including estimation results, the effects of changes in forces of attraction, and the effects of changes in choice sets. Section 5 demonstrates the extendibility of our model to estimate results comparable to existing literature, such as the status-merit exchange index, mate preferences due to age differences, odds ratios for education and age homogamy, the harmonic mean function approach, and transferable utility terms. Finally, Section 6 concludes.

1 Related Literature

The stochastic two-sided matching model we propose to employ originates from Dagsvik (2000), which is grounded in both random utility theory (McFadden 1984), and stable matching theory (Gale and Shapley 1962), with recent precedent literature by Menzel (2015) and Dagsvik and Jia (2022). The core concept is the stable matching where neither side of the match prefers being unmatched nor prefers other matches. Dagsvik (2000) has shown that by assuming that the observed matching is stable, there is a unique and stable equilibrium in the matching market under suitable conditions. Specifically, when the utilities are separable functions of a deterministic term and a random term, one can recover, from the data of matching, the joint surplus of matching that represents the forces of attraction of both sides toward the matching. We refine Dagsvik (2000)'s model to examine U.S. marriages, with a particular focus on the impacts of race, education, and age, and extend it to differentiate the impacts of changes in forces of attraction and partner availability.

Our analysis is closely related to several strands of literature. The first is the two-sided model proposed by Logan et al. (2008). In their study, Logan and colleagues estimate men's and women's mate preferences using a game-theoretic behavioral framework combined with a discrete-choice estimation method. Like them, we explicitly model both sides of the marriage market; however, we take a full structural approach, characterizing outcomes by the numbers and compositions of participants and by individual utilities, whereas they follow a more reduced-form strategy that infers preferences from observed patterns in relative characteristics (e.g., age differences among matched couples). When we apply our model to 2010 data, we recover a very similar pattern that Logan et al. (2008) found in 1988: men achieve their highest utility from partners who are two years younger, and women achieve their highest utility from partners who are two years older. In other words, the ideal-age gap between husband and wife remains to be two in 2010 as in 1988. And marriages with age difference that deviates from two are less attractive to both partners. Interestingly, compared to 1988, marriages where husband–wife age gap is less than two years are relatively more desirable in 2010, while marriages where the husband is older than wife by more than two years are less desirable.

The second strand of literature to which our model closely related involves the conditional logit model used to examine homogamy. Recent literature includes Qian and Lichter (2018), and Gullickson (2021, 2022). Similar to those articles, our model builds on the probability function of matches, which shares similarities with the conditional logit setup. However, while Qian and Lichter (2018), and Gullickson (2021, 2022) form the choice set by assuming fictional marriages and randomly drawing potential partners, we characterize marriages as the outcome of a stable matching process in a marriage market where being single is included as an option. Despite the differences in the modeling framework, we reach similar conclusion: education homogamy is strong, whereas age homogamy is comparatively weaker.

The third strand relates to the status-merit exchange. Early efforts primarily used log-linear analysis, treating marriage frequency (or the ratio between women and men across different characteristics) as the dependent variable and including a status-exchange parameter as an independent variable while controlling for other factors (e.g., Qian 1997; Rosenfeld 2005, 2010; Gullickson and Fu 2010; Kalmijn 2010; Schwartz, Zeng, and Xie 2016). The latter wave is represented by Xie and Dong (2021), who estimate an exchange index within a quasi-causal inference framework. The quasi-causal inference framework attempts to address the drawbacks of the log-linear approach but is still subject to analogous limitations: For example, the option of staying single is ignored thus treating the conditional probability of marriages as if it were unconditional. Furthermore, when adjusting the distribution of the status of the non-focal partner (white wives) in the treated group (intergroup marriages) to match that of the control group (intra-group marriages), their framework introduces bias by overlooking the constraints inherent in the endogenous choice sets. The distribution simulated does not feasibly reflect reality. For instance, "resampling in-group marriages to achieve equivalence of the non-focal spouses' social status" artificially assumes equal choice sets facing the focal partner (husbands), with no differences between black and white husbands. As a result, the quasi-causal inference approach could be biased. Our model provides an alternative approach by deriving the measure directly from the market primitives: the individual preferences of the participants. We find positive status-merit exchange effects for both 2010 and 2019, differing from assessments using the quasi-causal approach.

The fourth strand relates to modeling the forces of attraction between two genders in the marriage market using harmonic mean functions. Qian and Preston

(1993) were among the early efforts to analyze the availability and forces of attraction by age and education in American marriages using this method. Focusing on marriage trends from 1973, 1980, and 1988, Qian and Preston (1993) applied a multivariate analysis approach while measuring forces of attraction using the harmonic mean function and availability using the ratio of the predicted marriage rate in a later year (assuming the forces of attraction remains unchanged) to the actual marriage rate in an earlier year. Our model estimates the forces of attraction structurally, enables us to test changes over time statistically, and explicitly incorporating changes in the set of market participants in terms of both size and structure.

The last strand focuses on the availability of potential partners in certain groups, particularly men, in U.S. marriages. See studies by Lichter et al. (1992), Charles and Luoh (2010), and Mechoulan (2011). Charles and Luoh (2010) argue that higher men imprisonment rates in the United States have lowered the likelihood of women marrying, leading affected women to increase schooling and labor supply. Mechoulan (2011) supports this, showing that black men incarceration lowers the odds of black non-marital teenage fertility while increasing young black women's school attainment and early employment. With the capacity to differentiate the impacts of both forces of attraction and changes in choice sets on marriages, our model provides additional evidence. Specifically, it indicates that, considering incarceration mainly occurs among the younger, black, and lower-education group, the lower likelihood of marriage in this group of women is primarily driven by reduced forces of attraction for marriage, with the reduced partner availability playing a secondary role.

2 The Methodology

2.1 The Theoretical Model

We study a two-sided matching market with suppliers (men) and demanders (women) or vice versa. The key idea of the model is the concept of stable matching: Consider a supplier (s) and a demander (d) who are matched together but at least one of them, say s, would prefer to be single rather than being matched to the other. Then, this matching is said to be blocked by the unhappy agent s. Alternatively, consider a matching where s and d are not matched to one another but both prefer each other to their respective counterpart in the actual matching. Then, the pair (s, d) will be said to block the matching. We say that a matching is stable if it is not blocked by any unhappy agent or there is no pair that block the matching.

In particular, we model the marriage market as follows. In each year, every single man and woman faces a choice: match with someone from the opposite sex or remain single. Their utility for matching depends on (1) observable partner characteristics, (2) unobserved idiosyncratic shocks, and (3) a baseline utility of being single. Marriage occurs where both partners form a stable matching. By estimating the parameters that govern these utilities, we can infer how much of observed marriage-rate changes arise from shifts in "forces of attraction" versus "partner availability."

Assuming the marriage market consists of F women and M men, where individuals are divided into types based on observable characteristics. In our context, the types would be race, education, and age, respectively, or combined.[1]

Let F_i be the number of women of type i, M_j be the number of men of type j, $U^w_{si}(d,j)$ be the utility of woman s of type i for being matched to man d of type j, and $U^m_{dj}(s,i)$ be the utility of man d of type j for being matched to woman s of type i. Similarly, $U^w_{si}(0)$ is the utility of woman s of type i of being single (unmatched) and $U^m_{dj}(0)$ is the utility of man d of type j of being single. We assume that the utility functions have the structure as below:

$$U^w_{si}(d,j) = a_{ij}\, \varepsilon^w_{si}(d,j)/\kappa \quad \text{and} \quad U^m_{dj}(s,i) = b_{ij}\, \varepsilon^m_{dj}(s,i)/\kappa, \tag{1}$$

where κ is a positive term that depends on F, such that $\lim_{N\to\infty} \kappa/\sqrt{F} = 1$. The terms a_{ij} and b_{ij} are positive and deterministic, representing the average forces of attraction (utility) of j to i, and that of i to j, respectively. The terms $\varepsilon^w_{si}(d,j)$, $\varepsilon^m_{dj}(s,i)$, $U^w_{si}(0)$, and $U^m_{dj}(0)$ are positive and independent type I extreme-value distributed random variables, and their cumulative distribution functions can be expressed as follows:

$$\begin{aligned} P\left(\varepsilon^w_{si}(d,j) \le u\right) &= P\left(\varepsilon^m_{dj}(s,i) \le u\right) = P\left(U^w_{si}(0) \le u\right) = P\left(U^m_{dj}(0) \le u\right) \\ &= \exp(-1/u), \end{aligned} \tag{2}$$

for $u > 0$. These error terms represent unobserved heterogeneity in forces of attraction. They may capture both the effects of variables that are perfectly known to the individual agent and the effects of unpredictable fluctuations to the individual agent's tastes. The rationale for the fluctuations in tastes is that individuals may have difficulty evaluating the precise value of the different choices and may, therefore, revise their evaluations depending on their psychological state of mind.[2]

Furthermore, let $\varphi^w_i(j)$ and $\varphi^m_j(i)$ be the probabilities of a particular woman of type i to be matched with any man of type j and the probabilities of a particular man of type j to be matched with any woman of type i, respectively. And $\varphi^w_i(0)$ and $\varphi^m_j(0)$ represent the probability that a particular woman of type i and a particular man of type j remain single, which reflects how our model incorporates the choices of remaining single. Under the assumptions discussed above, as in Equation (1) and Equation (2), following the derivations of Dagsvik (2000), we obtain the probabilities of being matched or remaining single, as shown below:

$$\varphi^w_i(j) = \frac{M_j}{F} c_{ij}\varphi^w_i(0)\varphi^m_j(0), \quad \varphi^m_j(i) = \frac{F_i}{F} c_{ij}\varphi^w_i(0)\varphi^m_j(0) \tag{3}$$

$$\varphi^w_i(0) = \frac{1}{1+\sum_k c_{ik}\varphi^m_k(0)\frac{M_k}{F}}, \quad \varphi^m_j(0) = \frac{1}{1+\sum_k c_{kj}\varphi^w_k(0)\frac{F_k}{F}} \tag{4}$$

for all i and j, where $c_{ij} = a_{ij}b_{ij}$. As we lack prior knowledge regarding the relationship between a_{ij} and b_{ij}, we can estimate c_{ij} only without separately identifying a_{ij} and b_{ij}. The term c_{ij} is the key estimates used to numerically measure the average attractiveness (utilities) between a woman of type i and a man of type j, which is partially attributed to a_{ij} and partially attributed to b_{ij}.

However, separately estimating a_{ij} and b_{ij} would not yield additional insight into the matching process. Our objective is to recover the ordinal rankings of potential partners, and the composite term c_{ij} is sufficient for this purpose: for each woman of type i, c_{ij} captures her ranking of potential partners of type j, and analogously for each man of type j. Moreover, this interpretation is consistent with a symmetric Nash-bargaining framework closely related to the stable matching model of Gale and Shapley (1962). Although the deferred-acceptance algorithm is often described in terms of one side proposing and the other accepting or rejecting, stability requires that both parties weakly prefer the match to their outside options. Under the assumption of equal bargaining power, marital surplus is shared symmetrically, and the estimated affinity parameters c_{ij} are therefore best interpreted as measures of joint match compatibility (shared surplus), rather than as separately identified male and female preferences. This interpretation is consistent with empirical evidence suggesting that marriage choices reflect mutual interests and joint utility.

Because i and j can represent groups defined by combinations of observed characteristics, such as race, education, and age, the difference in c_{ij} between any values of i and j directly represents the difference in the forces of attraction among different potential group combinations. For instance $i_0 =$ {white, high school education, 25 years old}, $j_0 =$ {white, high school education, 25 years old}, and $j_1 =$ {black, high school education, 25 years old}, then the difference between $c_{i_0 j_0}$ and $c_{i_0 j_1}$ would represent the difference in attractiveness caused by the husband's race changing from white to black, conditional on both the woman and the man having high school education, being 25 years old, and the wife being white.

Note that the number and composition of participants on both sides of market, which we refer as the measure of "partner availability," are represented by the sizes of each type of agents F_is and M_js. From Equations (3) and (4), we see that these parameters influence directly the matching (marriage) probabilities.

The relations in (3) and (4) are in fact the asymptotic aggregate equilibrium conditions that must hold in every stable matching given the joint forces of attraction c_{ij}. Note also that (3) forms a nonlinear equation system in $\{\varphi_i^w(0), \varphi_j^m(0)\}$ and it follows from Dagsvik (2000) that there exists a unique solution for $\{\varphi_i^w(0), \varphi_j^m(0)\}$ as a function of the matching market primitives $\{c_{ij}, F_i/F, M_j/F\}$. Consequently, by (3) $\{\varphi_i^w(j), \varphi_j^m(i)\}$ can also be represented as functions of these parameters.

2.2 Estimation and the Maximum Likelihood Method

Let Y_{ij} denote the number of matched couples where the women are of type i and the men of type j, and let Y_{i0}^w and Y_{j0}^m represent the respective numbers of women of type i and men of type j who remain single at the end of the period. The probability that a particular (single) woman of type i and a particular (single) man of type j will be matched is equal to $\varphi_i^w(j)/M_j = \varphi_j^m(i)/F_i$.

Using Equation (3), it follows that the log-likelihood function of the data can be expressed as

$$\log L = \sum_i \sum_j Y_{ij} \log \left(\frac{\varphi_i^w(j)}{M_j} \right) + \sum_i Y_{i0}^w \log \varphi_i^w(0) + \sum_j Y_{j0}^m \log \varphi_j^m(0)$$
$$= \sum_i \sum_j Y_{ij} \log c_{ij} + \sum_i (Y_i + Y_{i0}^w) \log \varphi_i^w(0) + \sum_j (Y_j + Y_{j0}^m) \log \varphi_j^m(0) + K, \quad (5)$$

where $Y_i = \sum_j Y_{ij}$, $Y_j = \sum_i Y_{ij}$, and $K = \log F$ is a constant. The constant K will not affect the maximization results. Thus, we leave the constant K out in the following equations. Equation (5), together with (4), can be further transformed as

$$\log L = \sum_i \sum_j Y_{ij} \log c_{ij} + \sum_i F_i \log \varphi_i^w(0) + \sum_j M_j \log \varphi_j^m(0)$$
$$= \sum_i \sum_j Y_{ij} \log c_{ij} - \sum_i F_i \log \left(1 + \sum_k c_{ik} \varphi_k^m(0) \frac{M_k}{F} \right) + \sum_j M_j \log \varphi_j^m(0) \quad (6)$$

Due to the nonlinearity of the equilibrium relations in (4) it may be cumbersome to maximize the log-likelihood function in (6) by a direct approach. We propose instead an iterative approach to compute the likelihood function. To this end, it is convenient to reparameterize the force of attraction parameters by letting $v_{ij} = \log c_{ij}$. Let $v_{ij}(N)$ be the value of v_{ij} that corresponds to the Nth iteration (i.e., $v_{ij} = \lim_{N \to \infty} v_{ij}(N)$), and similarly let

$$\varphi_{i,N+1}^w(0) = \frac{1}{1 + \sum_k \exp(v_{ik}(N)) \varphi_{k,N}^m(0) \frac{M_k}{F}},$$
$$\varphi_{j,N+1}^m(0) = \frac{1}{1 + \sum_k \exp(v_{kj}(N)) \varphi_{k,N}^w(0) \frac{F_k}{F}} \quad (7)$$

for $N = 1, 2, \cdots$. As mentioned above, Dagsvik (2000) has proved that the equations in (7) have an unique solution. The log-likelihood function at the Nth stage is thus given by

$$\log L_N = \sum_i \sum_j Y_{ij} v_{ij}(N) + \sum_i F_i \log \varphi_{i,N}^w(0) + \sum_j M_j \log \varphi_{j,N}^m(0). \quad (8)$$

As a starting value, one can use

$$\varphi_{j,1}^m(0) = \frac{Y_{j0}^m}{M_j}. \quad (9)$$

Furthermore, expanding upon Equations (1) and (2), we can derive forces of attraction for remaining single or self-matched, represented by c_{i0} and c_{0j}. When c_{i0} and c_{0j} are equal to one, $U_{si}^w(0)$ and $U_{dj}^m(0)$ are influenced solely by positive, independently distributed type I extreme-value random errors. Accordingly, c_{ij} can be used to determine which match is preferable to remaining single from a deterministic standpoint.

Based on the estimated parameters $\hat{C} = [\hat{c}_{ij}]$ and the vectors representing the number of market participants of each type on both sides, denoted $\vec{F} = [F_i]$ for women and $\vec{M} = [M_j]$ for men, we can simulate the *expected* number of matches formed in the marriage market between type i women and type j men:

$$\hat{Y}_{ij} = Y_{ij}(\hat{C}, \vec{F}, \vec{M}),$$

where functions $Y_{ij}()$ are simply reformulations of Equations (3) and (4). Importantly, $\hat{Y}_{ij}$ depends not only on the estimated forces of attraction parameter $\hat{c}_{ij}$ and the group sizes F_i and M_j of their own types i and j but also on the full set of parameters in the system. This reflects the interconnected nature of the matching market, where competition occurs across all types on both sides.

2.3 Decomposing Changes Due to Forces of Attraction and Partner Availability

In this section, we derive equations to quantify the impact of changes in the force of attraction and the partner availability on marriages.

The changes in marriages observed between two time periods t_1 and t_2 can be attributed to two factors: shifts in the *joint* force of attraction c_{ij} and changes in partner availability: namely the relative abundance or scarcity of women of type i or men of type j (or equivalently the relative shares to the total number of women). We define changes in matches due to the partner availability as $\Delta Y_{\text{partner avail.}}$ and changes in matches due to shifts in the force of attraction as $\Delta Y_{\text{pref.}}$. The calculations are as follows:

$$\Delta Y_{\text{pref.}} = Y\left(\hat{C}(t_2), \vec{F}(t_2), \vec{M}(t_2)\right) - Y\left(\hat{C}(t_1), \vec{F}(t_2), \vec{M}(t_2)\right), \tag{10}$$

$$\Delta Y_{\text{partner avail.}} = Y\left(\hat{C}(t_2), \vec{F}(t_2), \vec{M}(t_2)\right) - Y\left(\hat{C}(t_2), \vec{F}(t_1), \vec{M}(t_1)\right), \tag{11}$$

where $\hat{C}(t_k)$, $\vec{F}(t_k)$, and $\vec{M}(t_k)$ represent the corresponding parameters for time period t_k, $k = 1, 2$. Alternatively, $\Delta Y_{\text{partner avail.}}$ can be expressed as the residual difference between matches in t_1 and t_2 that cannot be explained by ΔY_{pref}. Specifically,

$$\begin{aligned}\Delta Y_{\text{partner avail.}} &= \hat{Y}_{ij}(t_2) - \hat{Y}_{ij}(t_1) - \Delta Y_{\text{pref.}} \\ &= Y\left(\hat{C}(t_1), \vec{F}(t_2), \vec{M}(t_2)\right) - Y\left(\hat{C}(t_1), \vec{F}(t_1), \vec{M}(t_1)\right)\end{aligned} \tag{12}$$

These measures provide direct assessment of the impacts of shifts in the forces of attraction and changes in the partner availability.

2.4 Population at Risk for Marriage

To define the population at risk for marriage, we include all individuals observed in the ACS who are single, divorced, or widowed at the time of the survey, as well as those who report having married in the past 12 months. This construction captures the set of individuals who were at risk of entering a marriage at the beginning of the observation period. The ACS records marital status for individuals aged 15 and older, with ages extending to 95, and we retain the full age range available in the data.

We do not impose additional age restrictions for two reasons. At older ages, although marriages are relatively rare, individuals remain formally at risk of marriage; in our framework, such low-probability matches are reflected in thin cells and correspondingly low estimated forces of attraction rather than biasing the results. At younger ages, marriage eligibility varies across U.S. states, with some states permitting marriage below age 18 under parental or judicial consent, and individuals may marry outside their state of residence. Imposing ad hoc exclusions would therefore introduce inconsistency across jurisdictions. Moreover, the Census Bureau's own tabulations of marital status are reported for the population aged 15 and older[3], providing a natural benchmark for our choice. Taken together, these considerations motivate retaining the full ACS age range in defining the at-risk population.

3 Data

We use the 1 percent ACS data for 2010 and 2019, respectively, from the IPUMS data set to examine the shifts in marriage forces of attraction and changes in partner availability over the past few decades. The selection of 2019, instead of 2020, to represent the decade starting from 2020 is intended to mitigate the influence of Covid-19 and its associated mobility constraints on marriage dynamics.

Central to our analysis is a pivotal question present in both survey years: whether respondents married within the 12 months preceding the date of the interview. We exclusively include marriages reported within the past year when tallying marriage counts. This approach allows us to capture a snapshot of marriage forces of attraction and partner availability specific to that year. Diverging from the majority practice in much of the existing literature, which aggregates all reported marriages regardless of the year of occurrence, our methodology ensures a precise assessment of changes over time by avoiding the conflation of matching forces across different marriage years.

Our methodology lends itself to extensions for examining historical trends in marriage forces of attraction. However, due to the lack of data granularity in census data from earlier decades (i.e., 1990 and 2000) provided by IPUMS, particularly regarding the timing of marriages within a year, direct comparisons with our estimates for 2010 and 2019 are unfeasible. Consequently, our analysis focuses solely on the recent decade.

As illustrated in Equation (5), our model requires three data series: the count of women of type i and men of type j initiating match searching, and the count of matches formed within the year. We employ a three-step process to obtain these requisite series. First, we compute the count of unmatched women of type i and men of type j after the completion of the matching process by tallying respondents indicating their marital status at the time of the interview as never married/single, divorced, or widowed. Second, we determine the count of matches formed within the year by identifying respondents who reported marriage within the 12 months preceding the survey; this count considers only one instance per household if both spouses report marriage. To capture the characteristics of spouses, we use survey variables indicating spousal co-residence. Finally, we aggregate the count of marriages with the count of remaining unmatched individuals to derive the total number of individuals entering match searching.

Next, we define individual groups based on their education, race, and age. The choice of grouping reflects both the feasibility of optimizing the likelihood estimation approach and the need for comparability with existing literature. Education is dichotomized into "high school" (comprising individuals with education up to grade 12 or less) and "college" (including individuals with at least one year of college education). In the 2019 survey, 56 percent and 44 percent of individuals fall into these categories, respectively. Race is categorized into three groups: white (77 percent), black (9 percent), and others (14 percent). Age is stratified into three brackets: the first quartile, the last quartile, and the middle 50 percent of those married within the past year. Due to minor disparities in the gender-based age distribution, the age cutoffs differ slightly. For women, the cutoffs are younger than 24, equal to or older than 38, and those in between. Correspondingly, for men, the cutoffs are younger than 26, equal to or older than 42, and those in between.

We classify men and women into 18 ($2 \times 3 \times 3$) types defined by education, race, and age, which together generate an 18×18 two-sided matching structure with rows indexing men and columns indexing women (Table Table A.1). The same classification is applied to the 2010 ACS to ensure comparability of choice sets and estimated marriage patterns over time. Table Table A.1 reports the resulting unweighted counts of marriages and singles, whereas Table A.2 presents the corresponding weighted counts that approximate population totals. Several type pairs exhibit zero observed marriages in the data, for instance, matches between older, college-educated men of other races and high-school-educated black women. Because population weights amplify contrasts between zero and nonzero cells, potentially generating extreme parameter values, we rely on unweighted counts in estimation. Importantly, in a two-sided random utility framework, identification depends on the relative ordering of the forces of attraction across type pairs rather than their absolute magnitudes, so the presence of zero cells affects levels but not the substantive ranking of preferences across groups or years.

In summary, the 2019 survey encompasses 947,582 women and 885,999 men participating in match searching, with 17.523 marriages reported within the 12 months preceding the survey. Similarly, the 2010 survey comprises 898,831 women and 807,434 men engaging in match searching, with 17,554 marriages reported within the 12 months preceding the survey.[4]

Again, unlike previous studies that rely on marital status to infer match forces of attraction, our approach utilizes actual marriages occurring within the preceding 12 months. This methodology enables the precise estimation of forces of attraction for a specific year and facilitates a precise comparison across time periods. For instance, the forces of attraction for a 35-year-old couple marrying in 2009 may differ from those of an otherwise identical couple married in 2004, due to differences in their ages at the time of marriage and the choice set of partners they faced. Conventional analyses, however, often group all married couples together, failing to distinguish between such cases. As a result, the 2004 couple would be incorrectly treated as if their matching behavior reflects that of a 35-year-old couple in 2009, when in fact it corresponds to a 30-year-old couple in 2004. Hence, the use of marriage status data without differentiating the year of marriage can lead to distortions in the estimation of forces of attraction.

One key limitation in our data is that we cannot distinguish "not marrying" from "cohabitation," so essentially we treat these two states as the singular alternative state to forming a marital union. In contemporary U.S. society, cohabitation has become a significant and growing alternative. Our current model does not distinguish between remaining single and cohabiting. This means that the estimated parameter representing the preference for being 'unmatched' aggregates both remaining single and cohabiting. Consequently, the interpretation of changes in the estimated 'attraction to marriage' should acknowledge this aggregation. A decline in the estimated forces of attraction for marriage, for example, could reflect either a greater preference for remaining single or a greater preference for cohabitation as opposed to marriage.

Another limitation of our data is that it contains only limited individual characteristics. Individual heterogeneity may not be sufficiently accounted for. Although the model accounts for changes in the number of available partners within demographic groups (implicitly reflecting factors such as incarceration rates by reducing the total count of available men within certain groups, particularly younger black men with lower education), it does not explicitly incorporate individual characteristics like a history of incarceration as a factor directly influencing attraction beyond simply affecting the count of available partners in a group. A history of incarceration, even if an individual is not currently incarcerated, could affect their attractiveness in the marriage market. The model, in its current form, attributes changes in marriage numbers not explained by overall demographic shifts in the available pool to changes in preference or the model's structural components. We acknowledge the limitation of omitting such factors that affect individual attractiveness or preference.

Descriptive Patterns

Building on the marriage and single counts reported in Tables A.1 and A.2, we first present a set of unconditional descriptive statistics to facilitate comparison with the prior literature. As shown in Table 1, the share of interracial marriages increased from about 9.1 percent in 2010 to 11.4 percent in 2019, whereas the odds ratio of interracial marriage rose slightly in both unweighted and weighted data, indicating a modest decline in racial assortative mating. Educationally mixed marriages constitute a substantial share of unions in both years–approximately 28.5 percent (Table 2); although these prevalence rates are similar across weighting schemes, the odds ratios increase over time, suggesting a rise in the relative likelihood of cross-education marriages compared with educational homogamy.

Table 3 further reports the educational configuration of interracial marriages following the status-exchange framework of Merton (1941) and Davis (1941). In both years, most interracial marriages involve partners with the same level of education,

Table 1: Interracial and same-race marriages in the ACS, 2010 and 2019.

	Unweighted		Weighted	
	Count	Percent	Count	Percent
Panel A: ACS 2010				
Interracial marriages (white–non-white)	1,603	9.1	338,534	9.2
Same-race marriages	15,951	90.9	3,326,313	90.8
Total marriages	17,554	100	3,664,847	100
Odds ratio (interracial vs. same race)	0.02		0.02	
Panel B: ACS 2019				
Interracial marriages (white–non-white)	1,993	11.4	420,344	11.5
Same-race marriages	15,530	88.6	3,241,139	88.5
Total marriages	17,523	100	3,661,483	100
Odds ratio (interracial vs. same race)	0.03		0.02	

Notes: Interracial marriages are defined as unions between white and non-white spouses, where non-white includes black and other race categories. Percentages are calculated as shares of all marriages. Odds ratios are computed from a 2×2 cross-classification of husbands' and wives' race (white vs. non-white) as $(b \cdot c)/(a \cdot d)$, where a denotes white–white marriages, b white–non-white, c non-white–white, and d non-white–non-white. Weighted estimates use ACS person weights. Odds ratios are descriptive and unadjusted.

Table 2: Educationally mixed and homogamous marriages in the ACS, 2010 and 2019.

	Unweighted		Weighted	
	Count	Percent	Count	Percent
Panel A: ACS 2010				
Educationally mixed marriages (HS–College)	4,996	28.5	1,033,095	28.2
Educationally homogamous marriages	12,558	71.5	2,631,752	71.8
Total marriages	17,554	100	3,664,847	100
Odds ratio (educational mixing vs. homogamy)	0.16		0.16	
Panel B: ACS 2019				
Educationally mixed marriages (HS–College)	4,987	28.5	1,048,763	28.6
Educationally homogamous marriages	12,536	71.5	2,612,720	71.4
Total marriages	17,523	100	3,661,483	100
Odds ratio (educational mixing vs. homogamy)	0.18		0.18	

Notes: Educationally mixed marriages are defined as unions in which one spouse has a college degree (bachelor's or higher) and the other does not. Percentages are calculated as shares of all marriages. Odds ratios are computed from a 2×2 cross-classification of husbands' and wives' education (high school vs. college) as $(b \cdot c)/(a \cdot d)$, where a denotes high school–high school marriages, b high school–college, c college–high school, and d college–college. Weighted estimates use ACS person weights. Odds ratios are descriptive and unadjusted.

Table 3: Educational status configuration among interracial marriages, ACS 2010 and 2019.

	Unweighted		Weighted	
	Count	Percent	Count	Percent
Panel A: ACS 2010				
White wife–non-white husband				
White partner less educated than non-white	93	11.6	23,885	13.8
Same education	550	68.5	114,596	66.0
White partner more educated than non-white	160	19.9	34,962	20.2
White husband–non-white wife				
White partner less educated than non-white	135	16.9	27,207	16.5
Same education	569	71.1	118,512	71.8
White partner more educated than non-white	96	12.0	19,372	11.7
Panel B: ACS 2019				
White wife–non-white husband				
White partner less educated than non-white	87	9.8	17,989	9.4
Same education	617	69.7	133,412	70.0
White partner more educated than non-white	182	20.5	39,350	20.6
White husband–non-white wife				
White partner less educated than non-white	192	17.3	42,091	18.3
Same education	797	72.0	162,275	70.7
White partner more educated than non-white	118	10.7	25,227	11.0

Notes: Restricted to interracial marriages between white and non-white spouses. Education is coded as high school (less than college) and college (bachelor's degree or higher). For each gender configuration, "white partner more educated" means the white spouse has college, while the non-white spouse has high school; "white partner less educated" means the white spouse has high school, while the non-white spouse has college; and "same education" includes both high school or both college. Percentages are computed within each gender configuration. Weighted estimates use ACS person weights. Categories follow the status-exchange framework of (Merton, 1941) and (Davis, 1941).

whereas unions in which the white wife is more educated modestly outnumber those in which she is less educated; these differences are limited in magnitude and account for a relatively small share of interracial marriages. This seemingly "reverse" exchange pattern likely reflects the higher overall educational attainment of white women rather than compensatory matching per se, underscoring the importance of separating educational availability from the forces of attraction, as our model does, when calculating the exchange index (see Section 5). Finally, as shown in Table 4, median ages in our sample exceed the Census Bureau's median age at first marriage because the ACS measures the age distribution of individuals who married in the past 12 months, including those in second or later marriages; however, the differences are not substantial.

These descriptive tabulations are based on unconditional statistics. In the next step, we turn to our two-sided random utility matching model, which separates partner availability (the composition of the marriage market) from the forces of attraction (systematic preferences over partner types). This distinction matters for interpreting the prevalence of educationally mixed marriages and the corresponding

Table 4: Mean and median age at marriage in the ACS, 2010 and 2019.

	Unweighted		Weighted		Census Bureau
	Mean	Median	Mean	Median	Median Age at First Marriage
Panel A: ACS 2010					
Males	35.1	31	34.6	31	28.2
Females	32.6	29	32.0	29	26.1
Panel B: ACS 2019					
Males	36.2	32	35.3	32	29.8
Females	34.1	30	33.3	30	28.0

Notes: The table reports mean and median ages of currently married individuals observed in the ACS. Unweighted and weighted estimates are calculated from ACS microdata using person weights. The Census Bureau figures report the median age at first marriage, which incorporate projections of marriage timing and non-marriage and are therefore not directly comparable to the ACS-based measures of age among those currently married.

odds ratios in Table 2, as these descriptive measures cannot fully disentangle shifts in the educational distribution of potential spouses from changes in forces of attraction. Separately, when we assess status exchange within interracial marriages (Table 3), our model-based exchange index captures status exchange conditional on both the forces of attraction and the distribution of characteristics on each side of the market (see Section 5).

4 Empirical Results

4.1 Force of Attraction Estimates for 2010 and 2019

The force of attraction parameters c_{ij} between type i women and type j men are reported with their original estimated magnitudes scaled by a factor of 100 for easier interpretation. Recall that the magnitude of c_{ij} reflects only the cardinal ranking of preferences over potential partners; its absolute value has no intrinsic meaning because utilities are identified only up to affine transformations. To attach substantive interpretation, one must instead consider the implied matching probabilities derived from the c_{ij} parameters. A higher c_{ij} value indicates a stronger force of attraction. Figure A.1 presents the forces of attraction for men with high school education and Figure A.2 for men with college education. The first categorization, shown on the vertical axis, is for men, whereas the second, for women, includes three races (white, black, other races), two education levels (high school denoted as HS and college), and three age groups (younger, middle-aged, and older). We select several estimated c_{ij} values to display in Figure 1 for ease of reading and reference.

As shown in Figure 1, the highest force of attraction occurs in 2010 between middle-aged, college-educated men of other races and women from the same group, with a value of 548.1. The same match also exhibits the highest force of attraction in 2019, albeit at a lower level of 293.1. In both years, the second-highest force

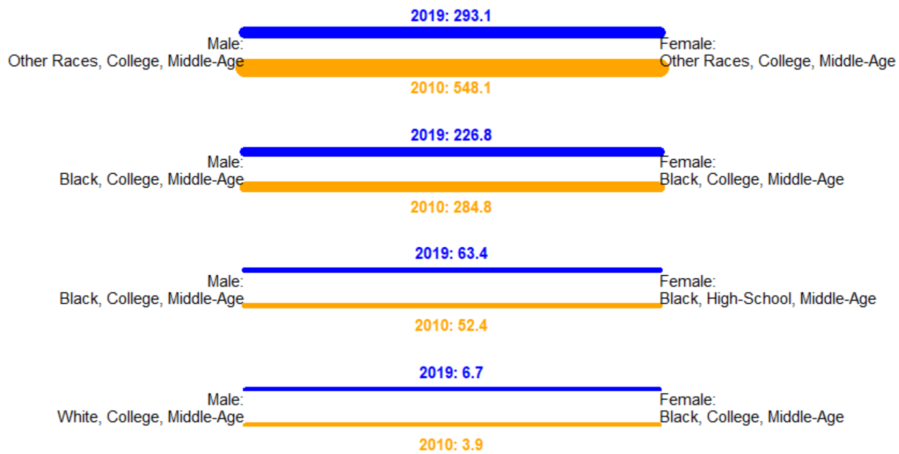

Figure 1: Estimated c_{ij} for selected matching groups.

of attraction is observed between middle-aged, college-educated black men and women, declining from 284.8 in 2010 to 226.8 in 2019.

Although attraction parameters are reported in cardinal units, the model is identified only up to monotonic transformations, implying an ordinal interpretation. Relative rankings across partner types are therefore meaningful, but absolute levels are not. As a result, differences in reported values–such as 548.1 versus 3.9 in 2010–should not be interpreted as reflecting proportional differences in attraction.

Although the force of attraction weakens for most groups between 2010 and 2019, it strengthens for a subset of matches. In particular, attraction increases for pairings between black, high-school-educated, middle-aged women and black, college-educated, middle-aged men, as well as between black, college-educated, middle-aged women and white, college-educated, middle-aged men, as indicated by thicker blue bars relative to orange bars in Figure 1. The strengthening of attraction for these matches may reflect gender differences in the growth of economic returns to higher education over this period. Among men with a bachelor's degree, median annual earnings increased by approximately $14,108 between 2010 and 2019, compared with substantially smaller gains for men whose highest credential is a high school diploma. A similar but smaller pattern holds for women: median annual earnings among college-educated women rose by roughly $8,860, whereas women with only a high school education experienced an increase of about $4,416. These differential earnings trajectories suggest that the rising labor-market premium to higher education increases the attractiveness of college-educated partners for women, while reducing the relative importance of partner education for men (see groups 2 and 3 in Fig. 1).[5]

Marriage within the other-race group exhibits the strongest force of attraction in both years, followed by within-group marriage among black individuals, as indicated by the longest bars in the right columns and the second-longest bars in the middle columns in Figures A.1 and A.2.

More details can be found in Table A.3 in the appendix. The columns in Table A.3 represent the force of attractions for women across the 18 categories. The rows of Table A.3 correspond to similar categories but for men. The value corresponds to the marriage where the wife's type indicated by the column and the husband's type indicated by the row. For example, in 2019, the force of attraction for a marriage between a black woman with college education in the middle-aged group and a man with the same characteristics is 226.8. Similarly, the force of attraction for a marriage between the same type of woman and a black man with high school education in the middle-aged group is 59.5.

If the diagonal parameters (highlighted in gray) are the maximum values in their respective rows and columns, it indicates that assortative matching by age, education, and race (i.e., homogamy) holds. That is, across all groups, partners who share same characteristics are the most attractive. For women, if the highest "force of attraction" in any row is not on the diagonal (i.e., not with men of the same group), which represents a deviation from assortative matching. Likewise, for men, any row whose maximum value lies off the diagonal indicates a deviation. We shade women's deviations in green and men's deviations in yellow.

For instance, in 2019, black middle-aged women with high school education showed a higher force of attraction for black middle-aged men with college education over those with matched college education. A similar pattern is observed for black middle-aged men with high school education, who preferred women with college education over those with only high school education.

Assortative matching–defined in our framework as higher utility assigned to match categories in which women and men share similar observable attributes (education, age group, and race)–is prevalent across most groups in 2010, but less so in 2019. In 2019, we observe four deviations from homogamy for men and three for women, compared with three for men and two for women in 2010. Although these patterns appear to differ from the well-documented rise in educational homogamy and decline in racial endogamy in the United States, the additional deviations observed in 2019 primarily occur within same-race matches that differ in education. Our findings therefore suggest that trends in homogamy may vary across dimensions: educational homogamy appears to weaken over time, whereas racial homogamy remains largely unchanged. By modeling these dimensions jointly, our framework captures such differential patterns. The results should be interpreted with caution, as they may be sensitive to the chosen group definitions.

In addition, we compare the estimated c_{ij} values to the "benchmark" of remaining single. By construction, the force of attraction for self-matches (c_{i0} and c_{0j}) is equal to one (scaled to 100 for comparability)[6]. When c_{ij} exceeds 100, it indicates that the deterministic part of the utility, that is, force of attraction, exceeds the utility of remaining single. Conversely, c_{ij} values lower than 100 suggest that the attractiveness of the match falls below that of remaining single, implying that any observed marriages in such cases can be attributed to random components in the forces of attraction.

As shown in Table A.3 in the appendix, in 2010, five groups exhibited forces of attraction exceeding 100, whereas in 2019, only four groups displayed forces of attraction higher than 100. The values in 2019 are notably smaller than those in 2010, indicating a substantial decrease in the attractiveness of marriage, which is consistent with the discussion above. Over the past few decades, a consistent trend shows that non-white individuals generally exhibit a higher force of attraction toward marriage. Middle-aged adults, particularly those with higher education, have the strongest marriage preference, surpassing younger cohorts. Nevertheless, the overall force of attraction toward marriage has declined during this period.

4.2 Changes of Force of Attraction between 2010 and 2019

One advantage of our method is that it allows us to formally test whether the c_{ij} significantly changed over time. For this purpose, we employ a likelihood-ratio test. When estimating c_{ij} separately for 2010 and 2019, the maximum log likelihoods are -304,841(2019) and -291,340 (2010). Under the null hypothesis that forces of attraction remain unchanged (i.e., $c_{ij}^{2010} = c_{ij}^{2019}$), the constrained model's maximum log likelihood is -597,586. The resulted likelihood ratio is 1,405.[7] With 324 parameters restricted, the 5 percent critical value for a chi-square distribution is approximately 367. Because our test statistics (1,405) far exceeds 367, we reject the null hypothesis and claim that c_{ij} values for 2019 are significantly different from those of 2010.

To illustrate the quantitative implications of changes in the forces of attraction, we conduct a "fictional marriage" simulation. Specifically, we hold the 2019 cohort of women and men fixed and apply the 2010 estimates of c_{ij} (as derived in Section 2). This exercise quantifies the number of marriages that would occur under 2010 attraction parameters given the 2019 population composition. We find that there are 7,238 fewer observed marriages in 2019 than in the corresponding counterfactual based on 2010 forces of attraction, suggesting a decline in overall marriage propensity over the past few decades (Table A.4). At the same time, certain groups exhibit stronger forces of attraction in 2019. For example, black, high-school-educated, middle-aged women and white, high-school-educated men in younger age groups display higher observed marriage counts toward selected partner types than in the counterfactual simulation.

Meanwhile, both white women and white men with college education in the middle-aged group experience the largest declines in marriage, consistent with a reduction in the forces of attraction. For example, observed marriages among white, college-educated, middle-aged men are 2,107 fewer than in the corresponding counterfactual, whereas the shortfall for women in the same group is 2,471.

More details can be found in Table A.4 in the appendix, which shows the differences in the number of matches between actual marriages in 2019 and the counterfactual marriages, assuming the 2010 forces of attraction. Positive changes, highlighted in red, indicate that the actual number is higher than simulated, whereas negative changes, shaded in green, suggest that the actual number is lower than simulated. The sum of differences, presented in the last column for men and the last row for women, reveals an aggregated impact on number of marriages due to changes in the forces of attraction between 2019 and 2010.

4.3 Impact of Partner Availability Changes between 2010 and 2019

Given that both the 2010 and 2019 surveys sampled 1 percent of the population, direct comparisons of women and men numbers are feasible. First, let us examine

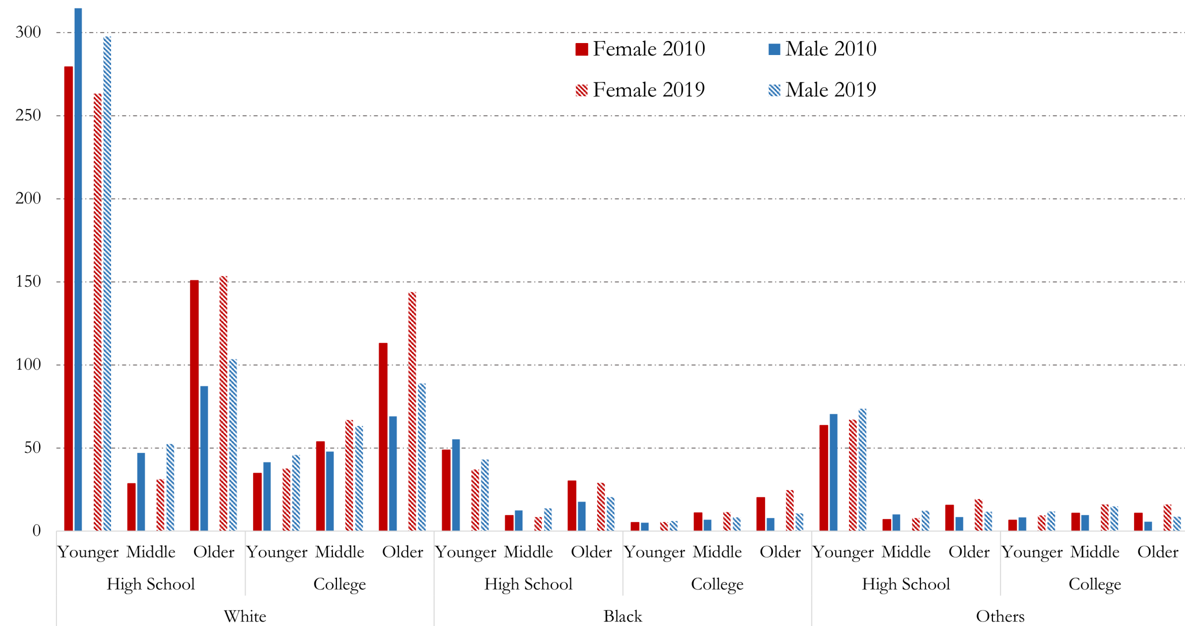

Figure 2: Number of available females and males: 2010 versus 2019 (in 1,000).

the changes in population size. Compared to 2010, the overall count of women entering the match-searching process increased by 5 percent, whereas the number of men increased by 7 percent in 2019. Figure 2 represents the counts of women and men across the 18 groups, with solid shading denoting 2010 and lined shading indicating 2019. Despite an overall increase in counts for 2019, the numbers of women and men in the younger age group with high school education, for both white and black races, declined: white women decreased by 6 percent compared to a 5 percent decline in white men, whereas black women decreased by 24 percent versus a 22 percent decline in black men. Notably, men of other races in the older-aged group with college education experienced the most substantial increase at around 51 percent, followed by women in the same group at approximately 56 percent.

As shown in Figure 3, across all younger groups, the ratios were higher than one, indicating more men than women, whereas all older groups exhibited ratios much lower than one, indicating more women than men. This reflects an imbalance in age structure, driven by demographic features of immigration and gender discrepancies in life expectancy. Middle-aged groups with lower education displayed ratios higher than one, indicating more men than women, whereas those with college education had ratios lower than one, indicating more women than men. This reflects that men with higher education are less available in the marriage market. Almost all groups experienced an increase in ratios (indicating more men) in 2019 compared to 2010, signifying an improvement in potential partners available to women. This is consistent with the higher increase in overall male match-searching participants: 7 percent compared to 5 percent.

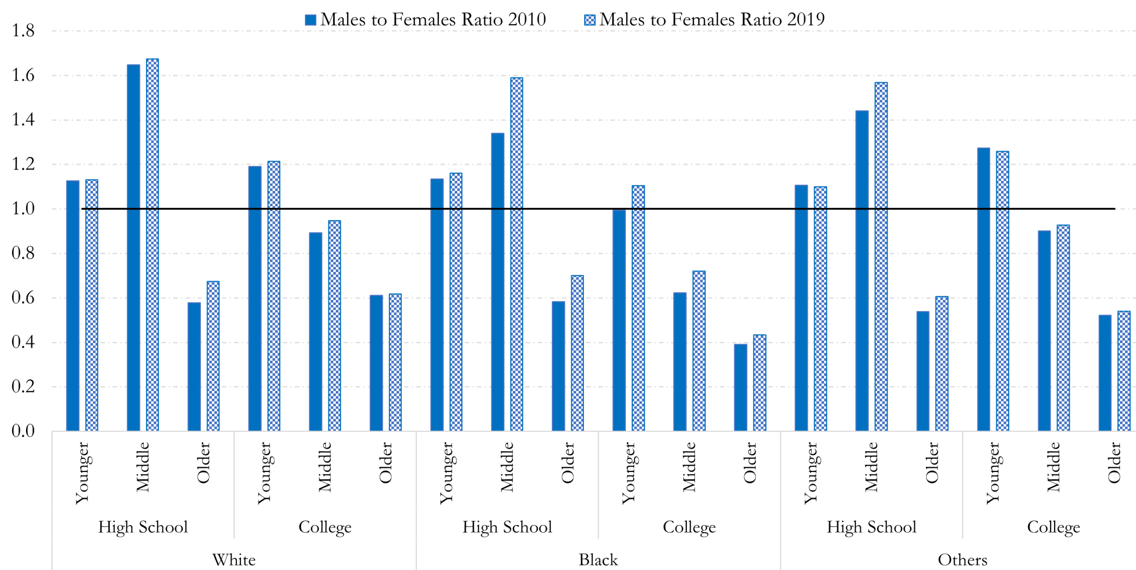

Figure 3: Ratio of available males to available females: 2010 versus 2019.

As discussed in Equation (12), the residual disparity in the number of marriages between 2019 and 2010, not accounted for by shifts in forces of attraction, can be attributed to changes in partner availability. This impact encompasses variations in both the absolute number of potential partners and structural shifts in the distribution of individuals across different groups. We find a net positive impact stemming from changes in partner availability from 2010 to 2019. Specifically, this results in 7,824 additional marriages, a notable 45 percent increase compared to the observed number of marriages in 2010 (Table A.5). This increase can be attributed to both a rise in the overall number of women and men entering match searching (5 percent and 7 percent, respectively) and an enhancement in the distribution across groups. The increase in marriages due to changes in availability surpasses the growth in available women and men, indicating a significant contribution from structural improvements. Across both genders, 16 out of the 18 groups experienced a positive impact from partner availabilities, with exceptions being white and black individuals in the younger age group with high school education–consistent with the observations presented in Figure 2.

Table A.5 in the appendix offers a detailed quantification of the marriage impact resulting from shifts in the numbers of women and men. Following Equation (12), the impact is computed as the disparity between the changes in the number of actual matches between 2019 and 2010 and the changes attributed to force-of-attraction changes (as shown in Table A.4). Positive changes are highlighted in red, whereas negative changes are shaded in green.

As shown in Table A.5, middle-aged white individuals with college education benefited the most from larger choice sets, resulting in 3,231 more marriages for women and 2,975 more marriages for men. The largest increase, of 2,182 more matches, occurred between women and men in the middle-aged group.

Conversely, black women and men in the younger age group with high school education encountered diminished availability. This finding partially resonates with existing literature, such as Charles and Luoh (2010) and Mechoulan (2011), which suggest that higher male imprisonment rates in the United States have diminished the likelihood of women getting married. However, our analysis reveals that two additional groups experienced even more restricted availability: white men and women with high school education in the younger age group. This divergence from existing literature underscores a significant nuance: it is not solely women but also men in the younger age group with lower education levels who confront reduced choice sets. Consequently, while higher male incarceration rates, particularly among black men, may contribute to the observed decline in marriages, they are just one factor among many shaping the evolving marriage landscape.

Meanwhile, black women in the younger age group with college education experienced a better partner availability. Although this group had fewer potential partner candidates among black men in the younger group with high school education, their enhanced availability partner candidates among black men in the middle-aged and older groups with high school education, particularly black men with college education, more than compensated for the reduced choice set of younger black men with high school education.

To further distinguish the effects of the number versus the distribution of potential partners on marriage formation, we simulate marriage outcomes assuming that the total number of women is as observed in 2019 but their distribution across groups matches that of 2010. These simulated matches are then compared with the actual matches in 2019; any disparity can be attributed solely to differences in composition between 2019 and 2010. The results are reported in Table A.6 in the appendix. Similarly, the impact of distributional changes among men is presented in Table A.7 in the appendix. A notable finding is that the change in women's distribution from 2010 to 2019 improved matching, resulting in 1,885 more marriages, whereas the change in men's distribution reduced matches by 2,079. Both of these effects are smaller than the 7,824 additional marriages observed when considering population size and distributional impacts across both genders.

5 Extending the Model for Comparability with Existing Literature

5.1 An Alternative Way to Construct the Exchange Index between Race and Education

Our approach can be used to calculate the exchange index between merit and status. Among recent work estimating the status-merit exchange, Xie and Dong (2021) estimated the exchange index between race and education using quasi-causal inference methods. They observe that when a white wife marries a black husband, the education level of the husband tends to be higher than that of white husbands. This pattern, inferred from the average education levels of husbands, raises an interesting question: could this be indicative of an exchange mechanism, wherein higher educational credentials are traded for racial differences? However, as extensively discussed in the literature, the observed differences in average education between the black and white husbands of white women may not have a casual interpretation due to the existence of potential confounding factors. For example, if white wives who marry black husbands have higher education themselves, the higher education levels observed in black husbands may not result from an exchange between race and education but rather from associative matching.

To deal with this potential bias, Xie and Dong (2021) propose a matching procedure, where they match interracial couples with intra-racial couples by adjusting for the education level of wives, based on the observed characteristics (education level in their example) of the wives. Under the widely assumed conditional independent assumption (CIA)– that, conditional on the observed characteristics, the "potential" education level of husbands is independent of their race or the education level of their wives (with interracial marriage being the treatment in Xie and Dong (2021)'s framework)–the matching procedure provides a casual estimator of the exchange index between race and education. Although this straightforward yet innovative approach offers new insights into studying status exchange in marriage, and despite its potential applications to similar problems in other matching markets, it remains subject to certain limitations.

One general drawback of the matching procedure is that it is often difficult, if not impossible, to completely justify the CIA assumption above. The same critique applies in this case as well. In principle, one would like to conditional on all factors that affect both the decision to enter interracial marriage and the "outcome" variable–in this case the education level of husbands. One possible such factor, as Xie and Dong (2021) pointed out, is the difference in the distribution of the nonfocal spouse's education status. However, the approach remains unclear whether one should control for this difference or not. Indeed, Xie and Dong (2021) acknowledge that "Clearly, whether to carry out this optional resampling step hinges on the researcher's null model of no status exchange."

Building on our two-sided matching model, we propose a different approach to calculate the exchange index. Our model can generate the counterfactual distribution of spousal education, thus the exchange index without invoking the CIA assumption and making assumption on the null model. However, the trade-off is reliance on the behavior assumptions underlying our two-sided matching model. We believe our approach offers an important alternative to that of Xie and Dong (2021). The choice of approach should be determined by researchers, based on which set of assumptions they believe are most likely to hold in their specific applications.

To facilitate comparisons, we first adopt the setups presented in Xie and Dong (2021) and briefly summarize their framework, where race is denoted by G (group) and education is indicated by S (status)

$$D = 1, \quad \text{if } G_{H_i} \neq G_{W_i}, \tag{13}$$

$$D = 0, \quad \text{if } G_{H_i} = G_{W_i}, \tag{14}$$

where G_{H_i} and G_{W_i} indicate the group in which the husband H and wife W belong to, that is, race, in marriage i. D is a dummy variable. When husband and wife in marriage i belong to different groups, the dummy variable D is equal to one; otherwise, D is equal to zero. Next, we differentiate the husband's status between intergroup and intragroup marriages

$$S_{H_i} = S^1_{H_i} \quad \text{if } D = 1, \tag{15}$$

$$S_{H_i} = S^0_{H_i} \quad \text{if } D = 0, \tag{16}$$

where S_{H_i} is the status, that is, education of the husband in marriage i. Note that when the husband and wife are belonging to the same race, indicated by $D = 0$, the husband status is equal to $S^0_{H_i}$; whereas the husband and wife are in different races, indicated by $D = 1$, the husband status is equal to $S^1_{H_i}$.

In Xie and Dong (2021), the so-called treatment effect of intergroup marriages on the treated (ATT) is

$$ATT(\delta_h|D = 1, G_W = 1) = E(S^1_H - S^0_H|D = 1, G_W = 1). \tag{17}$$

The treatment effect in Equation (17) is to measure, conditional on the wife is white ($G_W = 1$) and have non-white husband, the difference in education levels of her potential husbands, if she marry to a non-white ($G_{H_i} \neq G_{W_i}$) and to a white husband ($G_{H_i} = G_{W_i}$). The expected education level of a non-white husband can be calculated based on actual observations, whereas the expected counterfactual education level of a white husband can be calculated using the matching procedure

$$E(S^0_H|D = 1, G_W = 1) = \frac{1}{n_{01}} \sum_{i=1}^{n_{01}} S^0_{Hi*}, \tag{18}$$

where n_{01} denotes the number of the intergroup marriages ($G_H = 0, G_W = 1$) and S^0_{Hi*} represents the weighted average of husbands' education for the matched control cases, specifically the fictional intragroup marriages (white wives and white husbands) for marriage i. As previously discussed, S^0_{Hi*} is weighted to align the education distribution of wives in intragroup marriages with that of wives in intergroup marriages, thereby controlling for the impact of differing wives' education levels on husbands' education. This adjustment allows for isolating the remaining difference in husbands' education, attributing it solely to the influence of race.

Thus, the exchange index using the quasi-causal inference approach proposed by Xie and Dong (2021) is calculated as

$$EI_W(G_H = 0, G_W = 1) = \frac{1}{n_{01}} \sum_{i=1}^{n_{01}} (S^1_{Hi} - S^0_{Hi*}), \tag{19}$$

where S^1_{Hi} represents the observed education level of husbands in marriages between a non-white husbands and white wives.

Next, we illustrate how to use our approach to calculate the exchange index. In our framework, the distribution of a white wife's education level, whether married to a white husband or a non-white husband, can be derived directly from the two-sided matching model as follows.

For the treated case, that is when a white wife marries a non-white husband in marriage i, the probability of the husband's education level being x can be expressed as

$$P(S^1_{Hi} = x) = \frac{P(S_H = x, G_H \neq G_W, G_W = 1)}{P(G_H \neq G_W, G_W = 1)}, \tag{20}$$

where $P(G_H \neq G_W, G_W = 1)$ represents the probability of a white wife marrying a non-white husband, and $P(S_H = x, G_H \neq G_W, G_W = 1)$ represents the probability of a white wife marrying a non-white husband with education level x. In our data, the education level x takes two values: high school and below and college and above. This implies that the expectation of the potential education levels if the white wife marries a non-white husband in marriage i would be

$$E(S^1_{Hi}) = \sum_x x \cdot P(S^1_{Hi} = x) = \sum_x x \cdot \frac{P(S_{Hi} = x, G_H \neq G_W, G_W = 1)}{P(G_H \neq G_W, G_W = 1)}. \tag{21}$$

And similarly, for the non-treated case, that is, when a white wife marries a white husband in marriage i, the probability of the husband's education level being x can be expressed as

$$P(S^0_{Hi} = x) = \frac{P(S_{Hi} = x, G_H = G_W, G_W = 1)}{P(G_H = G_W, G_W = 1)}. \tag{22}$$

Thus, the exchange index, $EI_W(G_H = 0, G_W = 1)$ in Equation (19) would be expressed as the following using our approach:

$$\begin{aligned} EI_W(G_H = 0, G_W = 1) &= \frac{1}{n_{01}} \sum_{i=1}^{n_{01}} \left(E(S^1_{Hi}) - E(S^0_{Hi}) \right) \\ &= \frac{1}{n_{01}} \sum_{i=1}^{n_{01}} \left(\sum_x x \left(\frac{P(S_{Hi} = x, G_H \neq G_W, G_W = 1)}{P(G_H \neq G_W, G_W = 1)} - \frac{P(S_{Hi} = x, G_H = G_W, G_W = 1)}{P(G_H = G_W, G_W = 1)} \right) \right), \end{aligned} \tag{23}$$

where we sum over all white women who are observed to be married to a non-white spouse and take an average to obtain the ATT. Interestingly, because the expectations are the same for an individual and a group as long as they share the same characteristics–in this case, a white wife who marries a non-white husband or, had she married a white husband– Equation (23) is equivalent to the following:

$$\begin{aligned} EI_W(G_H = 0, G_W = 1) &= E(S^1_{Hi}) - E(S^0_{Hi}) \\ &= \sum_x x \left(\frac{P(S_{Hi} = x, G_H \neq G_W, G_W = 1)}{P(G_H \neq G_W, G_W = 1)} - \frac{P(S_{Hi} = x, G_H = G_W, G_W = 1)}{P(G_H = G_W, G_W = 1)} \right). \end{aligned} \tag{24}$$

Furthermore, it also can be calculated as follows:

$$\begin{aligned} EI_W(G_H = 0, G_W = 1) &= E(S^1_{Hi}) - E(S^0_{Hi}) \\ &= \frac{1}{n_{01}} \sum_{i=1}^{n_{01}} \sum_x x \cdot \frac{P(S_{Hi} = x, G_H \neq G_W, G_W = 1)}{P(G_H \neq G_W, G_W = 1)} - \frac{1}{n_{02}} \sum_{i=1}^{n_{02}} \sum_x x \cdot \frac{P(S_{Hi} = x, G_H = G_W, G_W = 1)}{P(G_H = G_W, G_W = 1)}. \end{aligned} \tag{25}$$

The flexibility demonstrated by the consistent results yielded by the three equations, as listed in Equations (23) to (25), highlights the advantages of our approach.

We continue the illustration using Equation (23). The probability functions of $P()$ in Equation (23) can be constructed based on Equations (3) and (4), which incorporate the choice of remaining single and the structures of choice sets. Specifically, utilizing Equations (3) and (4), we can derive the following:

$$P(S_{Hi} = x, G_H \neq G_W, G_W = 1) = \sum_{S_j = x, G_j = 0, G_i = 1} \frac{M_j}{F} c_{ij} \varphi_i^w(0) \varphi_j^m(0), \tag{26}$$

$$P(G_H \neq G_W, G_W = 1) = \sum_{G_j = 0, G_i = 1} \frac{M_j}{F} c_{ij} \varphi_i^w(0) \varphi_j^m(0), \tag{27}$$

$$P(S_{Hi} = x, G_H = G_W, G_W = 1) = \sum_{S_j = x, G_j = 1, G_i = 1} \frac{M_j}{F} c_{ij} \varphi_i^w(0) \varphi_j^m(0), \tag{28}$$

$$P(G_H = G_W, G_W = 1) = \sum_{G_j = 1, G_i = 1} \frac{M_j}{F} c_{ij} \varphi_i^w(0) \varphi_j^m(0). \tag{29}$$

Note that i in S_{Hi} on the left-hand side of the equations represents marriage i, i on the right-hand side of the equations indicates wife i, and j indicates husband. For example, the first equation requires that wife i is white ($G_i = 1$) and husband j is non-white with an education level at x ($S_j = x, G_j = 0$).

In sum, the difference between our approach and Xie and Dong (2021)'s method lies in how we calculate the expected education levels for both scenarios: when the white wife marries a non-white husband and when she marries a white husband. As shown in Equation (21), the expected education level of non-white husbands of white wives is different from the observed level, as we account for the conditional probability using our model.

Furthermore, as discussed above, our approach considers the possibility of remaining single and, more importantly, accounts for the differences in the distribution of characteristics among both wives and husbands of different races. This approach avoids the need to equalize the status distributions of the non-focal spouse (the white wives), as shown in Xie and Dong (2021)'s method.

To demonstrate the differences numerically, we calculate the exchange index using the naive approach,[8] the quasi-causal method proposed by Xie and Dong (2021) and our approach, based on data from the 2019 survey and the 2010 survey. The comparisons can be found in Table 3. If we rely solely on survey data rather than model-based estimates, for 2019, the average education of non-white husbands married to white wives is 12.45 years, compared with 12.74 years for white husbands married to white wives. In raw observations (the naive approach), non-white husbands have lower education levels than their white counterparts. This gives an estimate of the exchange index of -0.29 years, which is consistent with Table 5, using the case of white husbands married to white wives as the reference point. In contrast, the exchange index estimated using the quasi-causal method proposed by Xie and Dong (2021) is 0.20 years, whereas that estimated by our method is 0.62 year. In other words, Xie and Dong (2021) suggest a lower education level exchange of about 5 months than our estimates.

To contextualize the values of 0.62 years in terms of monetary value, we can utilize the annual earnings associated with different educational attainments.[9]

Table 5: Exchange Index Across Different Approaches.

	2019 Survey	2010 Survey
Naive	−0.29	−0.16
Quasi-causal	0.20	0.25
Our Method	0.62	0.52

Assuming a bachelor's degree takes four years and yields annual earnings of \$61,000, while high school completion results in annual earnings of \$39,000, the earnings difference per year is \$5,475. By multiplying this difference by 0.62 years, we calculate the premium based on our model to be 9 percent from the 2019 survey. These figures closely align with Wong (2003), who documented an income premium of 7 percent for intermarried black men.

For 2010, the exchange index estimated using our method is 0.52 years, compared with 0.25 years under the quasi-causal approach. The quasi-causal method therefore implies a decline in exchange between 2010 and 2019, whereas our approach indicates a strengthening over this period. In contrast, the naive approach suggests that non-white husbands have even lower education levels relative to their white counterparts in 2019 than in 2010.

5.2 Comparison with Logan et al.'s Estimation of Mate Preferences

Using the 1988 NSFH data, Logan et al. (2008) estimated how differences in age, education, and religion between the two genders affect forces of attraction, noting that these effects vary for men and women. Their study also builds on the stable matching framework established by Roth and Sotomayor (1990). However, unlike our study, Logan et al. (2008) do not make use of information on market participants to characterize the matching process.

Our approach can be used to produce the same measures as Logan et al. (2008), despite different modeling frameworks. For example, Figure 3 in Logan et al. (2008) shows how age differences between potential partners and the evaluating gender affect on attractiveness of a marriage. By categorizing age groups in the same way as Logan et al. (2008), we can calculate comparable estimates.

As shown in Table A.3 in the appendix, we represent male age groups using 26, 34 (the mean of 26-42), and 42 years, and female age groups using 24, 31 (the mean of 24-38), and 38 years. This categorization results in age differences for men of -18, -11, -10, -4, -3, -2, 4, 5, and 12 years, where the potential mates are women and the evaluating gender is man, and for women of -12, -5, -4, 2, 3, 4, 10, 11, and 18 years. Each cell in Table A.3 has an associated age difference, with the same absolute value but opposite sign for women and men. As c_{ij} listed in Table A.3 is the deterministic term of the utility function, as shown in Equations (1), (2), and (3), by taking the average of c_{ij}, conditional on the value of age differences, we produce measures comparable to Figure 3 in Logan et al. (2008).

We use 2010 survey data for these comparisons. As shown in Figure 4, the general pattern observed in Logan et al. (2008) using 1988 data remains in our analysis: men achieve the highest utility from partners who are two years younger,

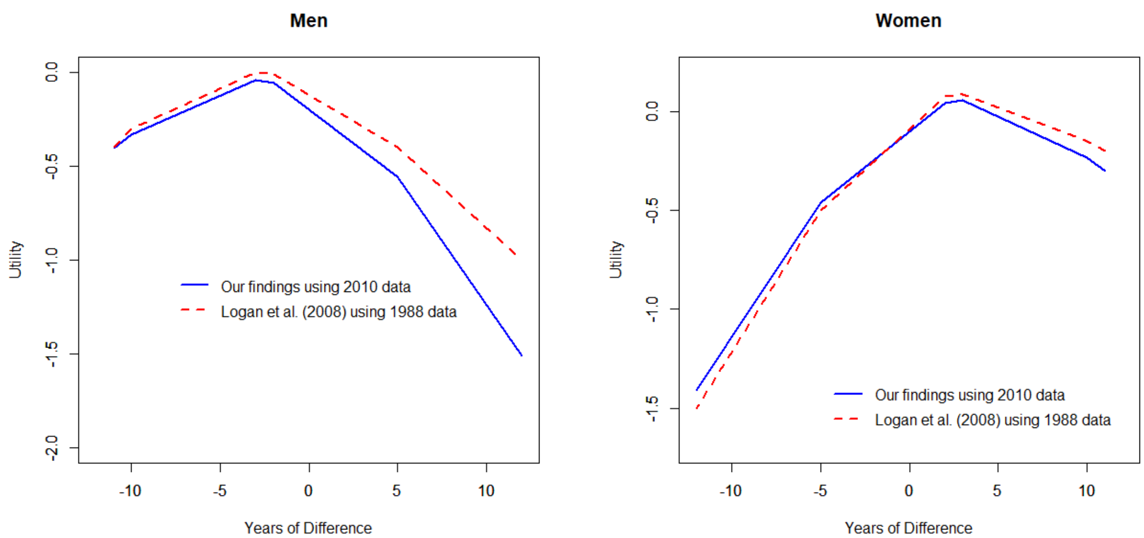

Figure 4: Impact of age difference on utility–a comparison with Logan et al. (2008). *Note:* Because the utility is cardinal without a tangible unit, we apply a logarithmic transformation twice and subtract 1 from our force of attraction estimates presented in Table A.3 to achieve a maximum utility for men similar to that depicted in Figure 3 of Logan et al. (2008). To address the additional volatility of our estimates, we exclude age differences of -4 and 4 for both genders. Finally, regarding the estimates of Logan et al. (2008), instead of plotting a polynomial curve, we opt to use the discrete y-values corresponding to the age differences, which explains why the line is not smooth, as presented in the original article.

whereas women achieve the highest utility from partners who are two years older. Relative to 1988, men's utilities in 2010 are lower for older partners and broadly similar for younger partners. In contrast, women's utilities in 2010 closely resemble those reported in Logan et al. (2008).

5.3 Comparison with Qian and Lichter's Analysis of Age and Education Homogamy

Our method also enables the estimation of odds ratio, a commonly used metric in the literature. For example, in the conditional logit framework of Qian and Lichter (2018), the odds ratio captures the likelihood of a match between two specific types of spouses relative to a reference match. Qian and Lichter (2018) found that among couples where both partners were in their first marriage, the odds of an actual marriage–relative to other potential matches (referred to as "fictional marriages" in their study)–were 68 percent lower when the wife was older than the husband, compared to the reference case where the husband was 0-2 years older than the wife (see column 2 of Table 6).

Our method allows for a straightforward calculation of odds ratios. Once the c_{ij} terms in Table A.3 are estimated, Equations 3 and 4 enable us to compute the probability that a man of type j matches with a woman of type i (and symmetrically, that a woman of type i matches with a man of type j), as well as the probability that each remains single. The odds ratio is then obtained by comparing the resulting match probability of interest with that of a chosen benchmark group.

Table 6: Odds ratios: age and education homogamy.

	Our Estimates Using 2010 Survey	Qian and Lichter (2018)'s Estimates Using 2008–2014 Pooled Survey Data*
Age differences		
Wife older	0.10	0.32
Husband 0–2		
Husband 3–5	0.14	0.50
Husband 6 or more years old	0.14	0.12
Education combination		
Same education		
Husband more educated	0.18	0.36
Wife more educated	0.26	0.50

*First-marriage husband and first-marriage wife.

Because Qian and Lichter (2018) used data from 2008 to 2014, we apply our estimates from the 2010 survey (Table A.3) to calculate a comparable odds ratio. We define the reference matches as those along the diagonal of Table A.3, where the husband is 0-2 years older than the wife. For marriages in which the wife is older, we find the odds ratio to be 90 percent lower than that of the reference matches (see column 1 of Table 6).

The odds ratio revealed by our analysis for husbands who are 3-5 years older is lower than that reported in Qian and Lichter (2018), by 36 percent, but is similar for husbands who are six or more years older. Several reasons may account for these differences. First, Qian and Lichter (2018) calculates odds ratios by comparing actual and fictional marriages, whereas we compute odds ratios relative to all other options, including marriage to any other spouse type or remaining single. Second, we use the 2010 national survey sample as the choice set, rather than locally available choices pooled from 2008 to 2014. Finally, the estimates reported in Qian and Lichter (2018) are restricted to first marriages, whereas our analysis does not impose this restriction. Taken together, our results indicate that when remaining single is included as a choice and the national survey sample is used as the choice set, age homogamy appears more prevalent.

The degree of educational homogamy observed in our analysis is stronger than that reported in Qian and Lichter (2018). Relative to marriages in which husbands and wives have the same level of education, the odds ratio for marriages in which the husband is more educated is 82 percent lower, and that for marriages in which the wife is more educated is 74 percent lower. In contrast, the corresponding reductions reported in Qian and Lichter (2018) are 64 percent and 50 percent, respectively.

5.4 Comparison with the Harmonic Mean Function of Marriage

Our model is also closely related to another important strand of literature on marriage, which builds on the harmonic mean function proposed by Schoen (1988); for instance, see Qian and Preston (1993) and Qian and Lichter (2007). In the harmonic mean function framework, the forces of attraction between women of type i and men of type j, which are conceptually similar to c_{ij}, take the following form:

$$\alpha_{ij} = \frac{Y_{ij}}{F_i M_j}\left(F_i + M_j\right), \tag{30}$$

where α_{ij} represents the force of attraction between women of type i and men of type j. Y_{ij} denotes the number of marriages between women of type i and men of type j in a specific time period, typically one year. F_i and M_j denote the number of available women and men of corresponding types in the middle of the year. The force of attraction α_{ij} can be estimated and utilized to predict marriages for other periods, as demonstrated by Qian and Lichter (2007).

The c_{ij} in our model shares similarities with the force of attraction, α_{ij}, when the sample size is large. Equation (3) in our analysis can be rearranged as follows:

$$c_{ij} = \frac{F_i \varphi_i^w(j) F}{F_i \varphi_i^w(0) \varphi_j^m(0) M_j}. \tag{31}$$

In a large population, one can readily estimate c_{ij} because $F_i \varphi_i^w(j)$ in Equation (31) is approximately equal to the number of married couples where the wives are of type i and the husbands are of type j, represented by Y_{ij}. $F_i \varphi_i^w(0)$ and $\varphi_j^m(0) M_j$ are approximately equal to the corresponding numbers of women and men who remain single at the end of the period. If we represent $F_i \varphi_i^w(0)$ and $\varphi_j^m(0) M_j$ by F_i' and M_j', Equation (31) can be expressed as

$$c_{ij} = \frac{Y_{ij}}{F_i' M_j'} F. \tag{32}$$

As demonstrated in Equation (30) and (32), our c_{ij} differs from α_{ij} in two aspects: first, c_{ij} has the corresponding numbers of women and men who remain single at the end of the period as the denominators, whereas α_{ij} has those that remain single in the middle of the period; second, c_{ij} adjusts a scale factor of total women number F that does not vary with the numbers of women of type i and men of type j, whereas α_{ij} adjusts a factor equal to the sum of numbers of available women and men of corresponding type in the middle of the year.

However, in small populations, one cannot use Equation (30) or (32) to estimate forces of attraction. In this case, we can employ Equations (3) and (4) because even though these relations are asymptotic, simulation experiments have shown that the asymptotic results yield quite close approximations in "small" populations. Furthermore, as demonstrated in Section 4, a key advantage of our approach–compared to the harmonic mean function method–is that it enables statistical testing of whether forces of attraction vary significantly across years.

5.5 Incorporating the Transferable Utility Framework of Choo and Siow

Finally, we discuss the connection between our model and the transferable utility framework proposed by Choo and Siow (2006). In this framework, upon matching, a part of the utility of one of the agents in the pair is transferred to the other to compensate for participation in the match. Dagsvik and Jia (2022) demonstrated that when the potential partners within each observational category are perfect substitutes and with suitable distributional assumptions of the error terms in the utility functions, the transfer model can be view as a special case of our model, with the utilities for the partners given by

$$U^w_{si}(d,j) = \tilde{a}_{ij}\exp(\omega_{ij})\varepsilon^w_{si}(d,j)/\kappa \quad \text{and} \quad U^m_{dj}(s,i) = \tilde{b}_{ij}\exp(-\omega_{ij})\varepsilon^m_{dj}(s,i)/\kappa. \tag{33}$$

The difference between Equation (33) and our baseline model in Equation (1) is the transfer item ω_{ij}, which can be understood as a transfer from woman of type i to man of type j. The transfer item, ω_{ij}, can be further expressed as

$$e^{\omega_{ij}} = \left(\frac{F_i\tilde{a}_{ij}\varphi^w_i(0)}{M_j\tilde{b}_{ij}\varphi^m_j(0)}\right)^{1/2}. \tag{34}$$

To estimate ω_{ij}, we require assumptions to enable the identification of $\tilde{a}_{ij}$ and $\tilde{b}_{ij}$. To illustrate, we assume that the forces of attraction are symmetric between women and men, such that $\tilde{a}_{ij} = \tilde{b}_{ij}$. Under this assumption, the transfer ω_{ij} can be simplified as

$$\omega_{ij} = \frac{1}{2}\Big(\log(F_i\varphi^w_i(0)) - \log(M_j\varphi^m_j(0))\Big). \tag{35}$$

The forces of attraction α_{ij} and b_{ij} in Equation (1) are comprised of the transfers of $\exp(\omega_{ij})$ and $\exp(-\omega_{ij})$ in Equation (33). In other words, $\alpha_{ij} = \tilde{a}_{ij}\exp(\omega_{ij})$ and $b_{ij} = \tilde{b}_{ij}\exp(-\omega_{ij})$. These transfers form a constituent part of the forces of attraction or attractiveness in our baseline model but with a specific direction. The direct relationship between our baseline c_{ij} and $\tilde{a}_{ij}$ (and $\tilde{b}_{ij}$) can be expressed as

$$\tilde{a}_{ij} = \frac{1}{\exp(\omega_{ij})}\cdot\sqrt{c_{ij}} \quad \text{and} \quad \tilde{b}_{ij} = \exp(\omega_{ij})\cdot\sqrt{c_{ij}}. \tag{36}$$

Using Equation (35) and the estimates of our baseline model, we can calculate the transfer item, ω_{ij}. Table A.8 in the appendix presents the transfer items estimated for 2019. The positive estimates of ω_{ij} indicate a transfer from women to men. Interestingly, variations in transfer amounts are observed only among men groups across rows of Table A.8. That is, women offer different transfers toward men with different characteristics. Conversely, men of the same characteristics receive the same transfer from women with different characteristics. The highest transfer of 6.2 is offered to young white men with high school education, followed by older white men with high school education at 6.0, and subsequently by older white men with college education at 5.9. The lowest offer of 4.3 is extended to younger black men with college education, followed by middle-aged black men with high school education at 4.5, and middle-aged men of other races with high school education at 4.5.

When analyzing changes from 2010 to 2019, as shown in Table A.9 in the appendix, most men groups experience an increase in their received transfer amounts in 2019 compared to those in 2010, except for young white men with high school education and black men with high school education across all age groups. The men of other races with college education have experienced the highest increase in transfers, approximately 5 percent.

6 Conclusion

This study introduces and applies a stochastic two-sided matching model, grounded in random utility theory and stable matching theory, to analyze marriage patterns. Using the observed marriage data in the United States in 2010 and 2019, we illustrate how this model can be used to disentangle the relative contributions of changes in preferences, captured as forces of attraction, and shifts in market participants, labeled as partner availability, to the evolving structure of marriage rates and sorting patterns over the past few decades.

The key parameters our framework are the joint attractiveness parameters (c_{ij}) across demographic combinations defined by age, education, and race. We show how these parameters can be estimated from typically available data on observed marriage formations. Based on these estimates, this framework enables a decomposition of the drivers of change in marriage behavior and allows for the computation of metrics comparable to those found in the existing literature, such as status exchange indices.

Our empirical analysis reveals systematic patterns of attraction across demographic groups and demonstrates how changes in observed marriage outcomes between 2010 and 2019 can be decomposed into preference-driven and supply driven components. For example, among young black women, the decline in marriage rates appears to be driven primarily by changes in the estimated preferences for marriage, whereas changes in the availability of potential partners played a more limited role. These findings contribute to a more nuanced understanding of the U.S. marriage market during this period. In particular, our results suggest that declining marriage rates are driven by reductions in the forces of attraction for marriage, whereas changes in the composition and size of the partner pool help counteract this reduction. This finding is especially pronounced in specific subgroups, underscoring the value of modeling preferences in a flexible, data-driven way.

An important limitation of the approach lies in its inability, without further assumptions, to separately identify men and women's preferences from observed matching data. In its current form, the model recovers only the joint matching surplus, or force of attractiveness between potential partners but not individual preferences for men and women. To achieve identification of individual specific preferences, it would require additional assumptions, such as assuming symmetric preferences or imposing homogeneity and functional form assumptions. Although this limits certain interpretive possibilities, we demonstrate that identification of the joint matching surplus c_{ij} is sufficient to address a broad set of relevant questions in marriage market analysis. Our results can be readily mapped onto and compared with findings from previous research, enabling reassessment of long-standing issues such as age and education homogamy, the role of demographic imbalance, et cetera. As discussed in the article, there are also some limitations with respect to our empirical analysis. For example, we are not able to distinguish between single and cohabitation. It is important to consider these concerns when interpreting our empirical findings.

Nevertheless, this study demonstrates that the stochastic two-sided matching model serves as a powerful and flexible tool for analyzing the marriage market. It contributes to a deeper understanding of the complex interplay between individual preferences and partner availability and provides a unified framework for interpreting changing patterns in marriage behavior.

Backmatter

Disclosure statements

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Notes

1 The population is sufficiently large so that properly defined asymptotic results will hold. We assume that the market is balanced in the sense that the ratios M/F, F_i/F, and M_j/M, tend toward positive finite constants, respectively, as F increases without bounds. This assumption is needed to guarantee that when population gets large, the market composition remains stable.

2 It is important that the error terms remain unchanged for a sufficiently long period to ensure that the matching algorithm is completed each time preferences are altered; otherwise, the matching may not be stable.

3 https://www.census.gov/data/tables/time-series/demo/families/marital.html

4 Tabulations exclude same-sex marriages and observations with missing partner information. Same-sex couples are included in the force-of-attraction estimations under an equal-probability gender assignment.

5 Median annual earnings are computed using publicly available statistics from the U.S. Bureau of Labor Statistics (BLS) and the College Board. For 2010, median weekly earnings of full-time workers ages 25 and older are taken from BLS (“Education Pays: Earnings

by educational attainment in 2010"). Weekly medians are converted to estimated annual earnings by multiplying by 52 weeks. This yields approximate 2010 annual earnings of: (1) men with a bachelor's degree: $1,171 × 52 ≈ $60,892; (2) women with a bachelor's degree: $920 × 52 ≈ $47,840; (3) men with a high-school diploma: $714 × 52 ≈ $37,128; and (4) women with a high-school diploma: $542 × 52 ≈ $28,184. For 2019, median annual earnings by educational attainment and gender are taken directly from the College Board's *Education Pays 2019* report (full-time, year-round workers), which reports: (1) men with a bachelor's degree: $75,200; (2) women with a bachelor's degree: $56,700; (3) men with a high-school diploma: $45,600; and (4) women with a high-school diploma: $32,600. Differences between 2010 and 2019 are computed by subtracting these 2010 annual estimates from the corresponding 2019 reported medians separately for men and women and for bachelor's degree versus high-school-diploma holders.

6 Because the utility of remaining single, along with the error terms entering into the utility of marrying (Equation 1), follows the same distribution as illustrated in Section 2, the preferences for self-matched are set equal to one.

7 $(-304841 - 291340) - (-597586) = 1405$.

8 The simple average observed for marriages between white wives and non-white husbands, and that of white wives and white husbands, in the data.

9 https://nces.ed.gov/programs/coe/indicator/cba/annual-earnings

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Yuan Cheng: Population Research Institute, Fudan University.
E-mail: chengyuan@fudan.edu.cn.
John K. Dagsvik: Research Department, Statistics Norway. E-mail: john.dagsvik@ssb.no.
Xuehui Han: Asia and Pacific Department, International Monetary Fund.
E-mail: XHan@imf.org.
Zhiyang Jia: Research Department, Statistics Norway. E-mail: Zhiyang.Jia@ssb.no.