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Supplement to:

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# Decoupling Inequality and Stratification in the American Wealth Distribution, 1989-2022: Online Supplement

## Appendix A: A Model of Cumulative Advantage and its Effects on Categorical Boundaries

To illustrate how cumulative advantage in wealth accumulation can generate divergence between inequality and stratification, we simulate a simple stochastic model of wealth dynamics. The model captures two key features emphasized in the literature: (1) expected returns to wealth increase with portfolio size, and (2) higher-return portfolios are also more volatile.

We consider a population of households indexed by $i$, whose wealth evolves according to:

$$W_{i,t+1} = W_{i,t} \exp\left(\mu(W_{i,t}) - \tfrac{1}{2}\sigma(W_{i,t})^2 + \sigma(W_{i,t})\varepsilon_{i,t}\right), \tag{1}$$

The functions $\mu(W)$ and $\sigma(W)$ govern the expected return and volatility of wealth, respectively, and are assumed to be increasing and saturating in wealth:

$$\mu(W) = \mu_0 + \alpha\left(1 - e^{-W/K}\right), \tag{2}$$

$$\sigma(W) = \sigma_0 + \beta\left(1 - e^{-W/H}\right). \tag{3}$$

The term $\varepsilon_{i,t} \sim \mathcal{N}(0,1)$ represents idiosyncratic shocks to returns. The adjustment term $-\frac{1}{2}\sigma(W_{i,t})^2$ ensures that $\mu(W)$ corresponds to the expected log return, preventing volatility from mechanically inflating expected wealth growth due to Jensen's inequality. In other words, it ensures that volatility only affects the variance of the realized rate of return, not its expectation. This specification is analogous to geometric Brownian motion (Øksendal 2003), and related formulations are widely used in models of inequality dynamics (e.g., Gabaix et al. 2016).

This formulation reflects the fact that wealth evolves multiplicatively over time, with returns proportional to current holdings. The exponential specification ensures that wealth remains strictly positive and allows returns to be modeled in log terms. Although net worth can take negative values in the real world, where it is possible to accrue debt, we restrict attention here to positive wealth levels for tractability. This simplification facilitates modeling proportional growth using log returns; incorporating negative wealth would primarily affect dynamics near the lower tail and does not substantively alter the mechanism of cumulative advantage emphasized here.

The model also abstracts from other features of real-world wealth accumulation, including income flows, savings behavior, and the intergenerational transmission of advantage. These factors are clearly important for understanding the full evolution of wealth distributions. However, the purpose of the simulation is not to reproduce the complete empirical process of wealth accumulation but rather to isolate a specific mechanism-wealth-dependent returns combined with stochastic shocks within a fixed population-and assess its implications for the relationship between inequality and stratification. By combining wealth-dependent expected returns with increasing dispersion, the model generates a setting in which inequality can rise through the stretching of the upper tail while rank positions across groups become less tightly aligned with group membership. In particular, the model is designed to evaluate whether cumulative advantage operating through returns on existing wealth can, by itself, produce increasing dispersion in wealth levels alongside decreasing predictability of rank by group membership. If such divergence arises in this stylized setting, it establishes that inequality and stratification do not mechanically move together, even under familiar and widely studied dynamics of wealth accumulation.

In the simulations presented below, we set $\mu_0 = 0.005$, $\alpha = 0.05$, and $K = 15$, implying a baseline return with a modest wealth-dependent premium that saturates at higher levels of wealth. For volatility, we set $\sigma_0 = 0.04$, $\beta = 0.16$, and $H = 10$, generating increasing but bounded exposure to return variability as wealth rises. We initialize two groups, $A$ and $B$, with lognormally distributed wealth. Specifically, initial wealth is drawn from lognormal distributions with parameters $(\mu_A, \sigma) = (2.0, 0.5)$ and $(\mu_B, \sigma) = (1.6, 0.5)$, so that group $A$ begins with a higher mean level of wealth than group $B$, while both groups have comparable dispersion and substantial overlap in their initial distributions. Each group contains 1,000 households. We then simulate the evolution of wealth over 30 periods according to the process described above. At each time step, we compute the difference in mean wealth between groups, the difference in median wealth between groups, and the rank-based stratification index $S$ described in Section 4.1 of the main text.

Figure 5 shows the evolution of these quantities over time. The simulation produces increasing gaps in both mean and median wealth between groups, reflecting the effects of cumulative advantage. At the same time, the stratification index declines, indicating increasing overlap between groups in rank terms. This divergence arises because higher-return, higher-volatility dynamics amplify dispersion within groups. As wealth grows, households in both groups increasingly access similar return regimes, while lower-wealth members of the advantaged group remain near the bottom of the distribution. As a result, group membership becomes a less reliable predictor of relative position, even as differences in central tendencies widen.

It does not appear that this result – declining stratification alongside widening between-group disparities in wealth levels – is particularly sensitive to the chosen values of the return gradient $\alpha$ and the volatility gradient $\beta$, as the figure below shows.

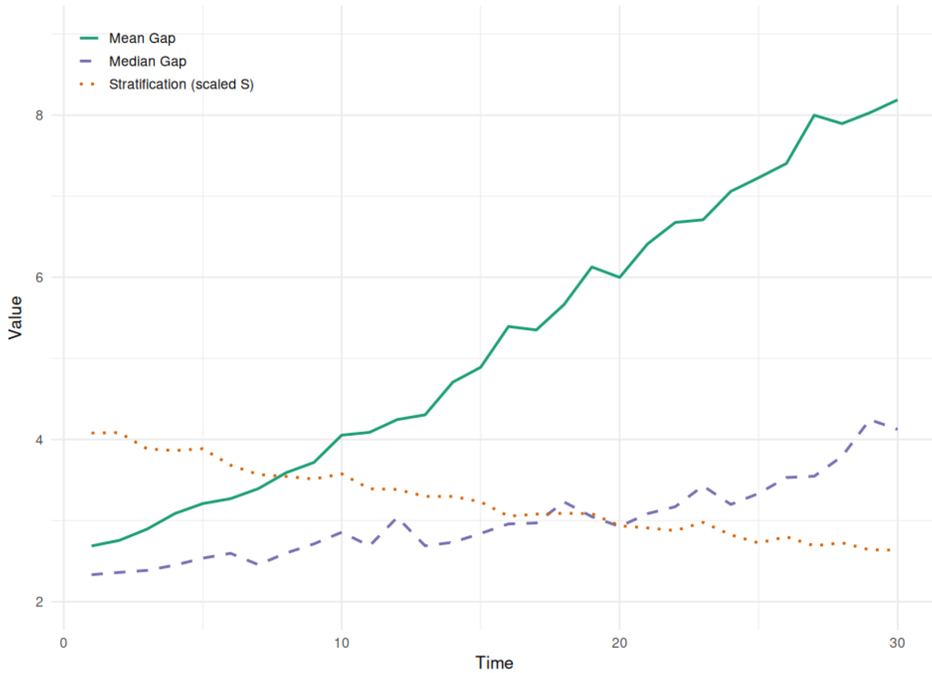


Figure 5: Simulation of cumulative advantage dynamics. The figure shows the evolution of the mean wealth gap, median wealth gap, and the rank-based stratification index $S$ (scaled by 10 for visibility) over time. While mean and median gaps increase, stratification declines due to increasing overlap in wealth ranks across groups.

The change in the stratification metric over 30 time steps ($\Delta S$) is variously but almost uniformly negative across a wide range of $\alpha$ and $\beta$, while the gap between the mean wealth levels of the two groups ($\Delta$ Mean Gap) strictly increases over the same range.

While the baseline specification assumes that households with similar levels of wealth face similar return processes regardless of group membership, this assumption may be too strong in some empirical contexts. In particular, it is plausible that households in disadvantaged groups face barriers to accessing high-return assets, favorable credit terms, or financial intermediation at low and moderate levels of wealth, even if these barriers diminish as wealth increases. To capture this possibility, we extend the model to allow for group-specific differences in expected returns that attenuate with wealth. Specifically, we modify the return

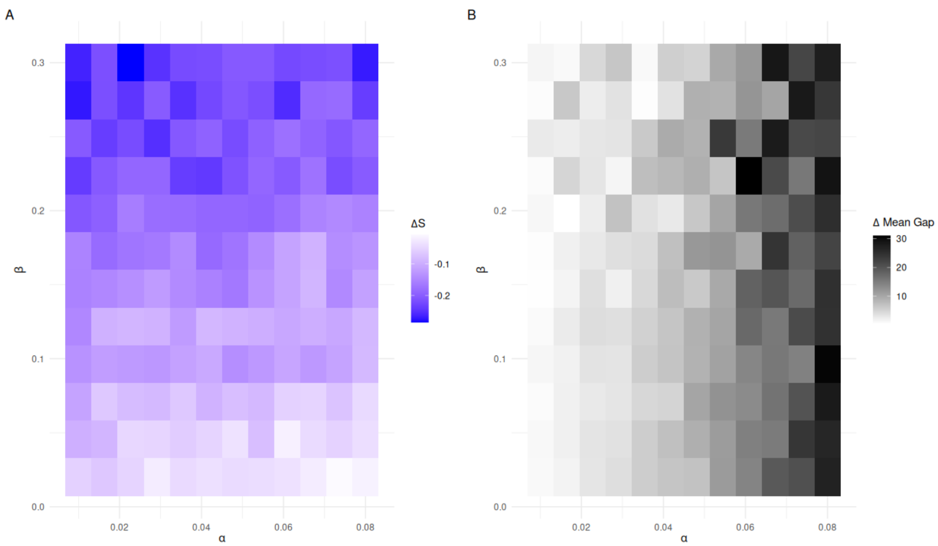


Figure 6: Change in stratification metric $S$ (panel A) and mean wealth gap between the two groups (panel B) over 30 time steps and across a range of return and volatility gradients.

function for group $B$ to include a wealth-dependent penalty:

$$\mu_A(W) = \mu_0 + \alpha\left(1 - e^{-W/K}\right), \tag{4}$$

$$\mu_B(W) = \mu_0 + \alpha\left(1 - e^{-W/K}\right) - \rho e^{-W/L}, \tag{5}$$

where $\rho > 0$ governs the magnitude of the disadvantage at low levels of wealth and $L > 0$ determines the rate at which this disadvantage attenuates. As $W \to \infty$, the penalty term $e^{-W/L} \to 0$, and the expected returns of the two groups converge. This formulation captures the idea that group-based barriers to wealth accumulation are most consequential at low and moderate wealth levels, but become less relevant as households accumulate sufficient wealth to access similar investment opportunities.

All other aspects of the model remain unchanged. In particular, wealth continues to evolve according to the multiplicative process in Equation (1), and volatility is governed by the same function $\sigma(W)$ for both groups. In the simulations presented below, we fix

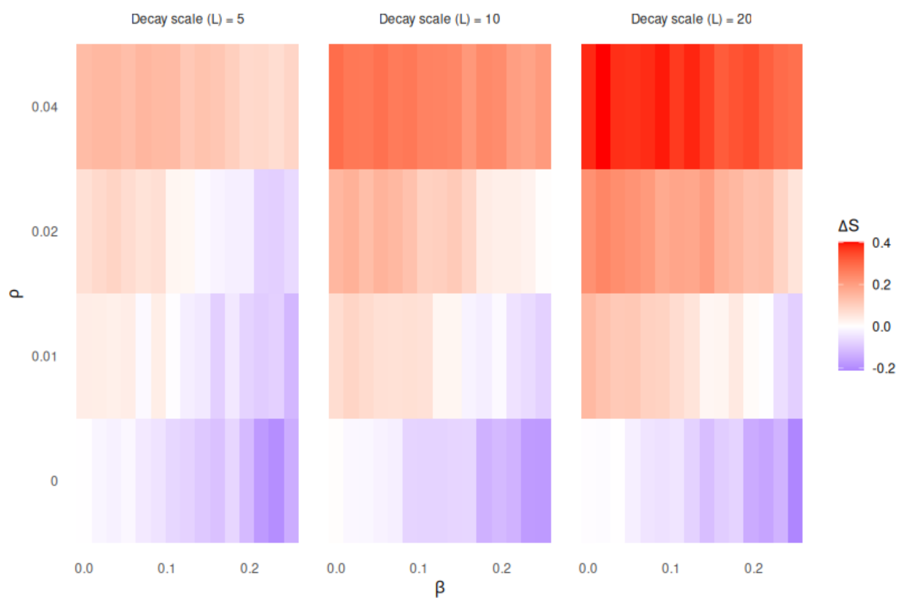


Figure 7: Change in the stratification metric $S$ over 30 time steps across values of the volatility gradient $\beta$, penalty parameter $\rho$, and decay scale $L$. Negative values indicate increasing overlap in wealth ranks across groups.

$\mu_0 = 0.005$, $\alpha = 0.06$, and $K = 15$, and set $\sigma_0 = 0.01$ and $H = 10$. We then vary the volatility gradient $\beta$ over the interval $[0, 0.25]$, the penalty parameter $\rho$ over $\{0, 0.01, 0.02, 0.04\}$, and the decay scale $L$ over $\{5, 10, 20\}$. Initial wealth distributions, group sizes, and the number of simulation periods are the same as in the baseline model.

Figures 7 and 8 show the change in the stratification index ($\Delta S$) and the change in the mean wealth gap ($\Delta$ Mean Gap), respectively, over 30 time steps across this parameter space. Across nearly all combinations of $\beta$, $\rho$, and $L$, the mean wealth gap increases. At the same time, the change in $S$ is negative across a substantial region of the parameter space, particularly when the penalty $\rho$ is modest or when it attenuates relatively quickly (smaller $L$).

These results indicate that the divergence between inequality and stratification is not limited to the baseline case in which groups differ only in their initial wealth distributions.

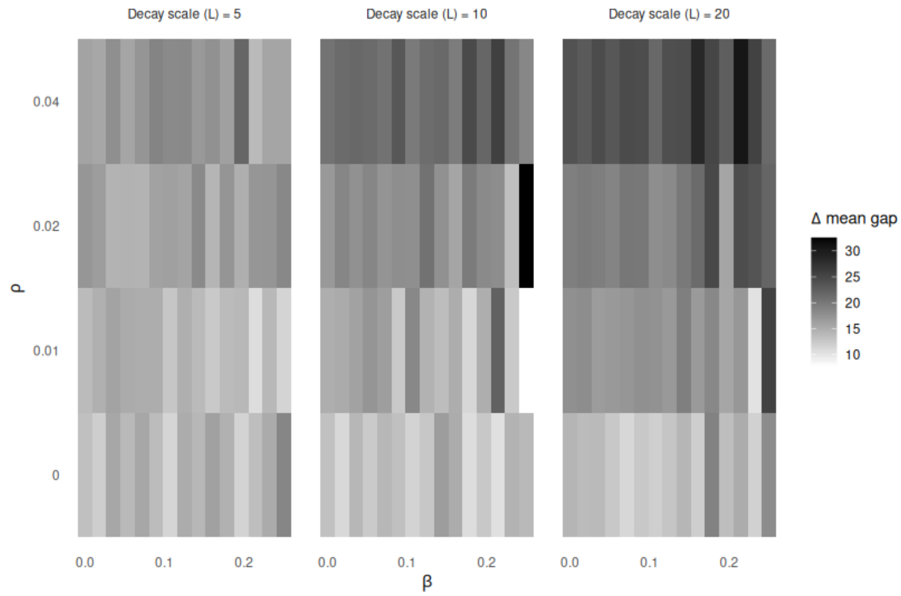


Figure 8: Change in the mean wealth gap between groups over 30 time steps across values of the volatility gradient $\beta$, penalty parameter $\rho$, and decay scale $L$.

Even when the disadvantaged group faces lower expected returns at low and moderate wealth levels, cumulative advantage combined with increasing within-group dispersion can generate greater overlap in wealth ranks across groups. In this sense, the mechanism linking rising inequality to declining stratification is robust to the introduction of countervailing forces that tend to harden group-based distinctions in wealth accumulation.

# Appendix B: Housing and Non-housing Wealth Dispersion

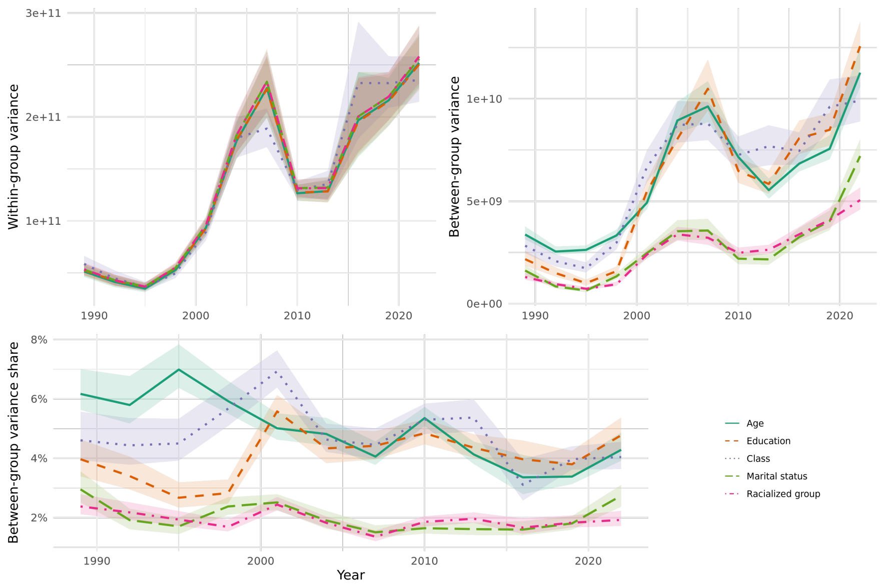


Figure 9: Change in within- and between-group variance in housing wealth over time, as well as change in the between-group share of total variance. Confidence bands are 95% and constructed via bootstrap sampling with 500 replicates.

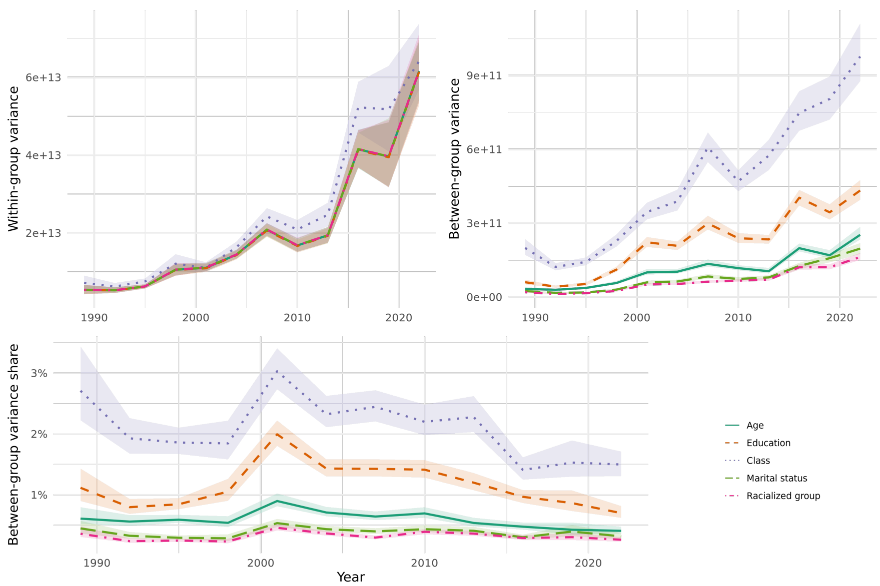


Figure 10: Change in within- and between-group variance in non-housing wealth over time, as well as change in the between-group share of total variance. Confidence bands are 95% and constructed via bootstrap sampling with 500 replicates.

# Appendix C: Coefficient of Mean Difference Over Time

The variance decomposition presented in Figure 2 documents that both within- and between-group wealth dispersion increased over the survey period. However, the variance is sensitive to the shape of the wealth distribution in ways that make it difficult to interpret as a measure of the magnitude of differences between specific pairs of groups: because wealth distributions are highly right-skewed, variance decompositions are heavily influenced by a small number of extreme values in the upper tail, and the between-group share of total variance does not have a straightforward interpretation as the expected difference between a randomly selected member of one group and a randomly selected member of another. Similarly, the rank-based stratification index $S$ asks only whether a member of a higher-ranked group tends to *rank above* a member of a lower-ranked group in pairwise comparisons, without taking into account how far apart their wealth levels are. If winning by more matters — that is, if the magnitude of the gap between groups is substantively important alongside its direction — then a complementary measure is needed.

The coefficient of mean group difference addresses this by preserving the pairwise logic of $S$ while incorporating the magnitude of differences.[1] Whereas $S$ records the expected sign of $Y_i - Y_j$ for cross-group pairs, the coefficient of mean group difference records the expected magnitude of that difference. Applied between groups, it measures the magnitude of categorical inequality; applied within groups, it measures the magnitude of inequality among members of the same category.

For two groups $a$ and $b$ with group means $\bar{Y}_a$ and $\bar{Y}_b$ and overall mean $\bar{Y}$, the between-group coefficient of mean difference is defined as

$$D_{ab} = \frac{\bar{Y}_a - \bar{Y}_b}{\bar{Y}}. \tag{6}$$

[1] We thank the deputy editor for suggesting this measure.

This measure expresses the mean wealth gap between groups as a multiple of overall mean wealth and is directly interpretable: $D_{ab} = 1$ means the mean gap equals overall mean wealth. Because net worth decomposes additively into housing wealth $H$ and non-housing wealth $N$, $D_{ab}$ decomposes as

$$D_{ab} = \frac{\bar{H}_a - \bar{H}_b}{\bar{Y}} + \frac{\bar{N}_a - \bar{N}_b}{\bar{Y}}, \tag{7}$$

where each term captures the contribution of a wealth component to the overall mean group difference, scaled by overall mean wealth.

The within-group coefficient of mean difference for group $g$ is defined as the expected absolute pairwise difference among members of the same group, relative to the group mean:

$$D_g^{\text{within}} = \frac{E(|Y_i - Y_j| \mid C_i = C_j = g)}{\bar{Y}_g}. \tag{8}$$

This captures the magnitude of wealth inequality within a group. A complementary between-group absolute measure is defined as

$$D_{ab}^{\text{between}} = \frac{E(|Y_i - Y_j| \mid C_i = a, C_j = b)}{\bar{Y}}, \tag{9}$$

which captures the expected magnitude of wealth differences between a randomly selected member of group $a$ and a randomly selected member of group $b$.

Since these measures are only defined for two groups, we simply demonstrate their utility using Black and White households as an example. All statistics are computed within each survey implicate and averaged across implicates following Rubin's rules, and bootstrapped 95% confidence intervals are constructed by case resampling within each implicate.

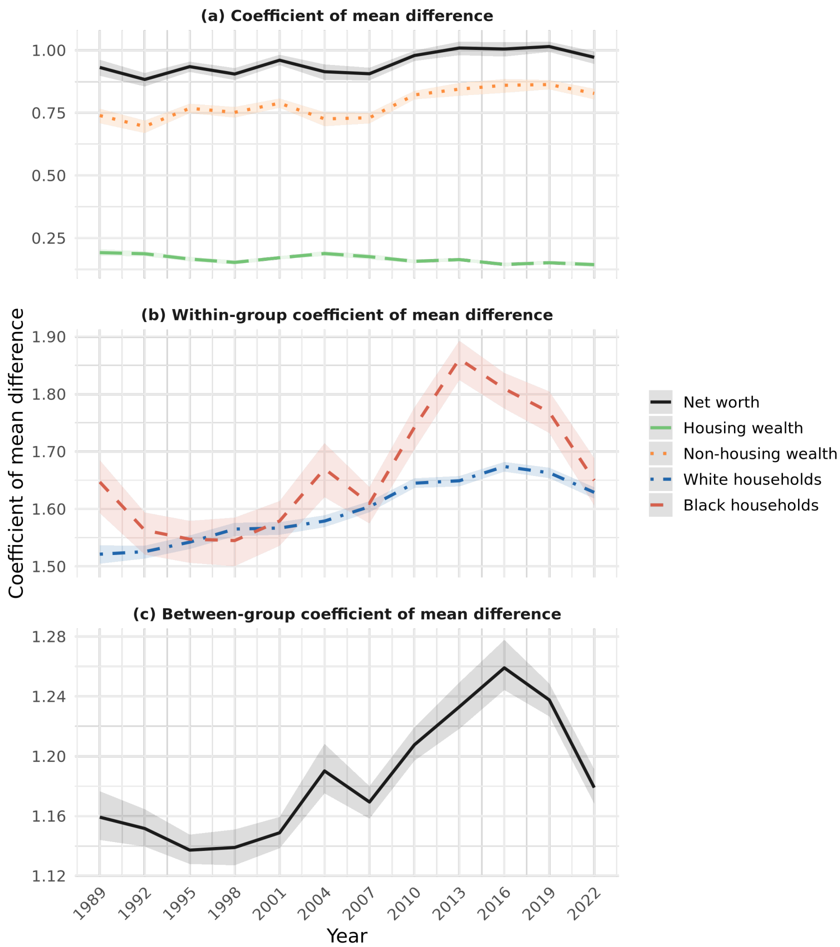


Figure 11: Coefficient of mean group difference measures for White and Black households, 1989–2022. Panel (a) shows the between-group coefficient of mean difference $D_{ab}$ and its decomposition into housing and non-housing wealth components. Panel (b) shows the within-group coefficient of mean difference $D_g^{\text{within}}$ for White and Black households separately. Panel (c) shows the between-group mean absolute difference $D_{ab}^{\text{between}}$. Shaded bands are bootstrapped 95% confidence intervals based on 500 replicates.

Panel (a) shows that the overall coefficient of mean difference grew modestly over the period, from $D_{ab} \approx 0.93$ in 1989 to $D_{ab} \approx 0.97$ in 2022, with a peak around 1.02 in the late 2010s. This growth was driven almost entirely by the non-housing wealth component. Panel (b) shows that within-group wealth dispersion, measured as the expected absolute pairwise difference relative to the group mean, is large for both groups throughout the period and exhibits substantial year-to-year variation, particularly among Black households. For White households, the within-group coefficient of mean difference increases steadily until 2016, after which it declines slightly. Panel (c) shows that the between-group coefficient of mean difference grew substantially over the period, from approximately 1.16 in 1989 to approximately 1.18 in 2022, with a notable peak around 2016.

Taken together, these results confirm that the magnitude of the White-Black wealth gap increased over this period, driven primarily by non-housing wealth. This is consistent with, and complementary to, the decoupling phenomenon elaborated in the main analysis: while the rank-based stratification index $S$ declined — indicating that a randomly selected Black household became more likely to outrank a randomly selected White household in wealth over time — the expected magnitude of the gap between a randomly selected pair of Black and White households grew.

# Appendix D: Stratification under Different Subdivisions of the Class, Racialized Groups, and Education Axes

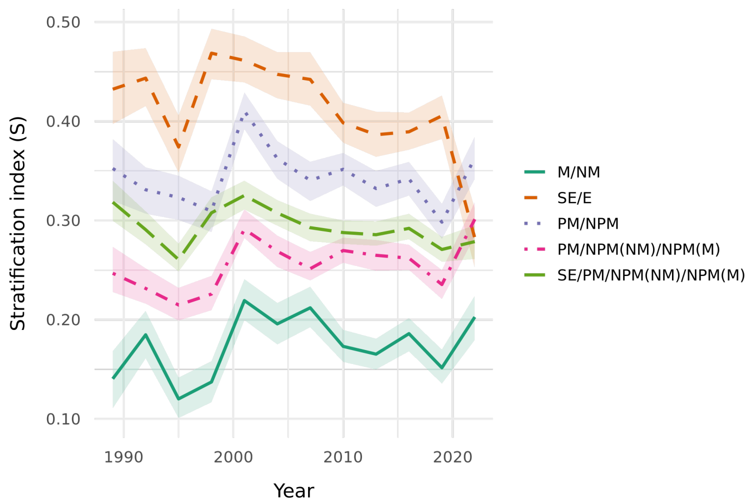


Figure 12: Change in net worth $S$ over time for different subdivisions of the class axis. Confidence bands are 95% and constructed via bootstrap sampling with 500 replicates. M = manual, NM = non-manual, SE = self-employed, E = employee, PM = professional/managerial, NPM = non-professional non-managerial, NPM(NM) = non-professional non-managerial (non-manual), NPM(M) = non-professional non-managerial (manual). For subdivisions that distinguish the self-employed category, all other categories consist strictly of employees. When the self-employed category is not distinguished, all other categories contain both self-employed respondents and employees.

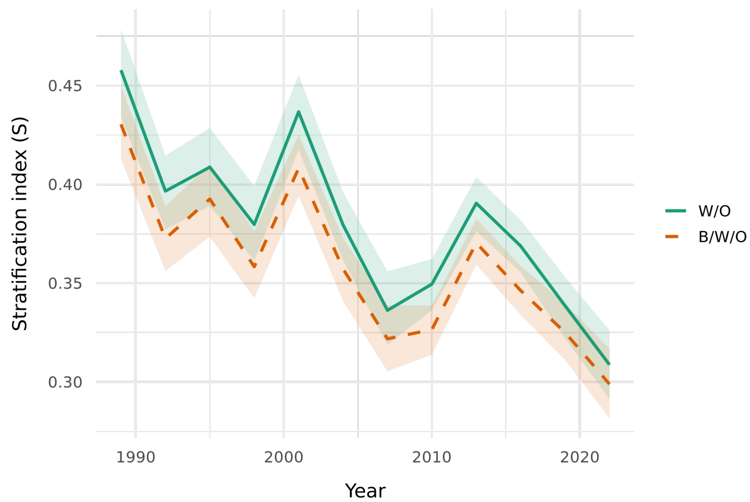


Figure 13: Change in net worth $S$ over time for different subdivisions of the racialized groups axis. Confidence bands are 95% and constructed via bootstrap sampling with 500 replicates. W = White, B = Black, O = Other.

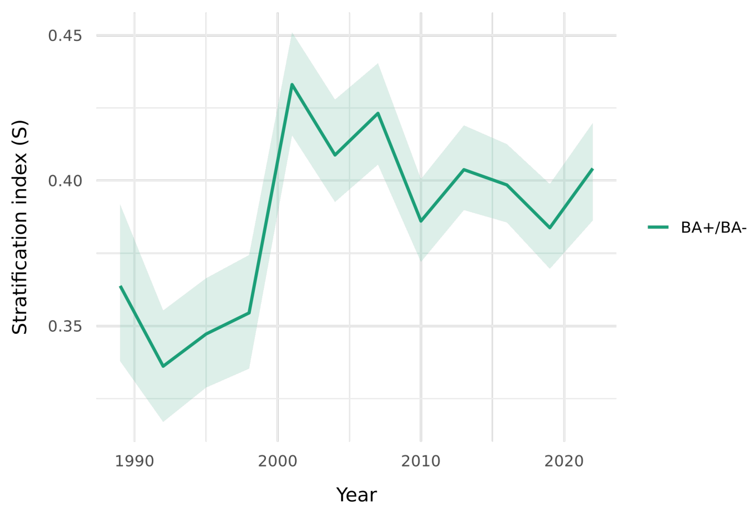


Figure 14: Change in net worth $S$ over time for different subdivisions of the education axis. Confidence bands are 95% and constructed via bootstrap sampling with 500 replicates. BA+ = bachelor's degree or more, BA- = less than bachelor's degree.

# Appendix E: Housing and Non-housing Wealth Stratification

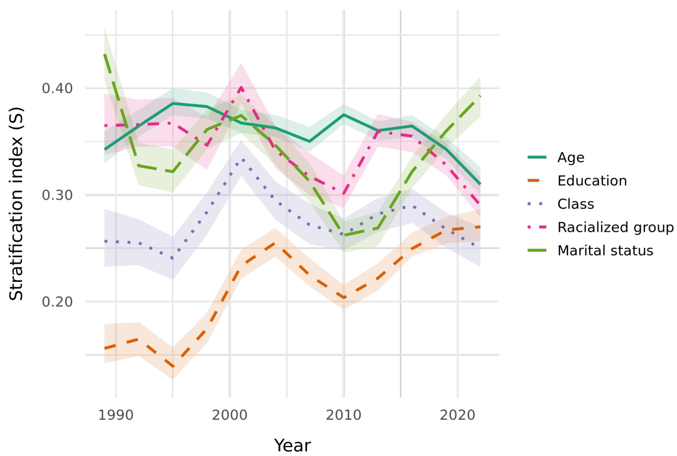


Figure 15: Change in housing wealth $S$ over time. Confidence bands are 95% and constructed via bootstrap sampling with 500 replicates.

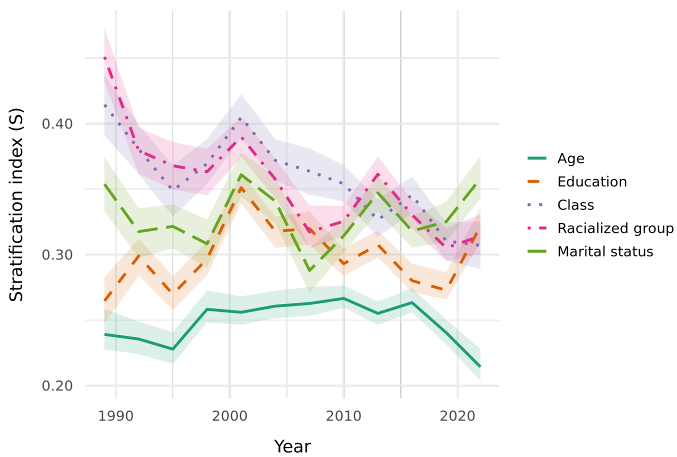


Figure 16: Change in non-housing wealth $S$ over time. Confidence bands are 95% and constructed via bootstrap sampling with 500 replicates.

# References

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