

Supplement to:

Henriksen, Lasse Folke, Jacob Aagard Lunding, Christoph Houman Ellersgaard, Anton Grau Larsen. 2025. "The Hardcore Brokers: Core-Periphery Structure and Political Representation in Denmark's Corporate Elite Network" *Sociological Science* 12: 769-803.

Appendices

K-core sensitivity checks

We ran a series of checks to test the sensitivity of the k -core decomposition to company sample restrictions. First, we randomly sampled bundles of companies from 90 percent to 10 percent of the total sample and ran our decomposition procedure. These tests show that our analyses are very sensitive to even a small set of randomly excluded companies. Results begin to change significantly already in the 80 percent bundle and at 50 percent there is no discernible core-periphery structure in the network, meaning that the k -core decomposition fails to identify a k -core set in the largest component of local brokers. Second, we repeated this procedure, but sampled bundles according to company rank, beginning with the 90 percent highest ranked companies and ending with the 10 percent highest ranked companies (see below for details on the ranking method). Again, we found results to change significantly, though more incrementally (see Figure A1). Third, we excluded the highest ranked 1000 companies in 50 company rank increments. These analyses show that the k -core decomposition is surprisingly resilient to top company exclusion. While for some observation months results are highly sensitive, most months yield a similar core size as the full sample population. This shows that the maximal k -core does not critically rely on the connections they forge through top company interlocks, because these directors become connected at smaller boards too (see Figure A2).

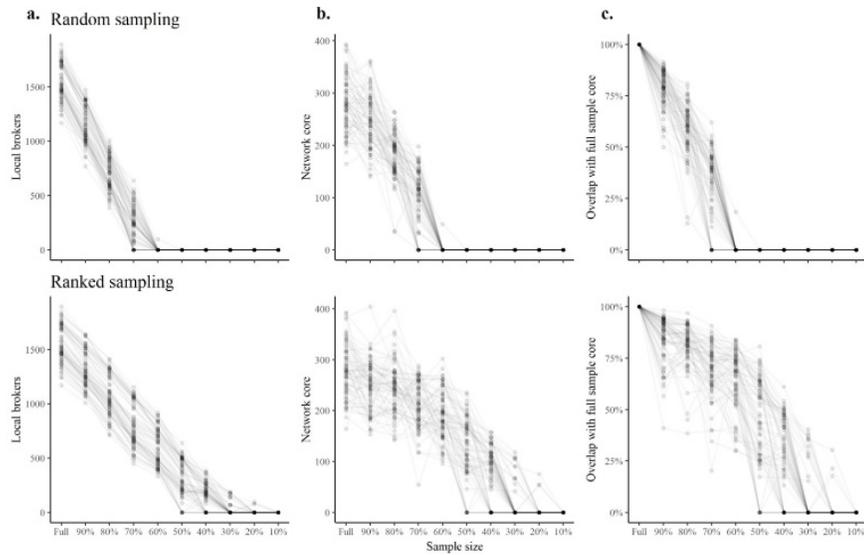


Figure A1. Consequences of company sampling.

Note: The figure plots the consequences of company sampling on the results of the local brokerage pruning and k-core decomposition for each month in the period 2010 to 2015. Each point represents a month, and the lines connect monthly points at different sample cuts. Starting from the full sample, we successively decrease the sample size (in steps of 10 percent) and report the number of local brokers and the network core (panel a and b). Panel c plots the percentages overlap with the network core in the full sample. In the upper row companies are sampled according to a randomly generated integer, while the lower panel row plots a sampling by company rank.

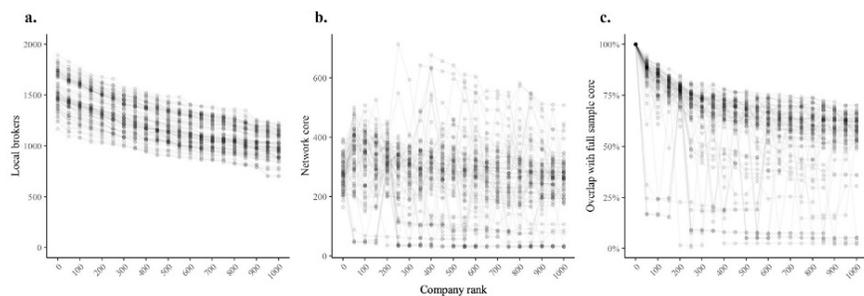


Figure A2. Consequences of large company removal.

Note: The figure plots the consequences of disregarding large companies from the sample on the local brokerage pruning and k-core decomposition for each month in the period 2010 to 2015. Each point represents a month and the lines connect monthly points at different sample cuts. Starting from the full sample, we successively exclude a greater number of companies (in steps of 50) starting from the largest, and report the number of local brokers (panel a) and the network core (panel b), and finally the overlap between the network core identified in the full and the limited sample (panel c).

We also tested how sensitive the k-core decomposition is to varying the local brokerage threshold described above. For the main analyses we sampled directors with a ratio equal to or above one in the local brokerage threshold, that is, directors with a local betweenness at least equal to their degree. We varied this ratio between 0.7 and 1.2 and ran the k-core decomposition at each threshold. We show that minor variations in the local brokerage threshold somewhat change the size and composition of the k-cores. Nevertheless, a threshold of one detects the most consistent core size across the months, suggesting a stable core of local brokers (see Figure A3). Although the size and composition of the k-cores vary somewhat when we change the local brokerage threshold, the statistical association between interlocker coreness and political characteristics remain similar to what we obtain with a threshold of one (see Figure A4).

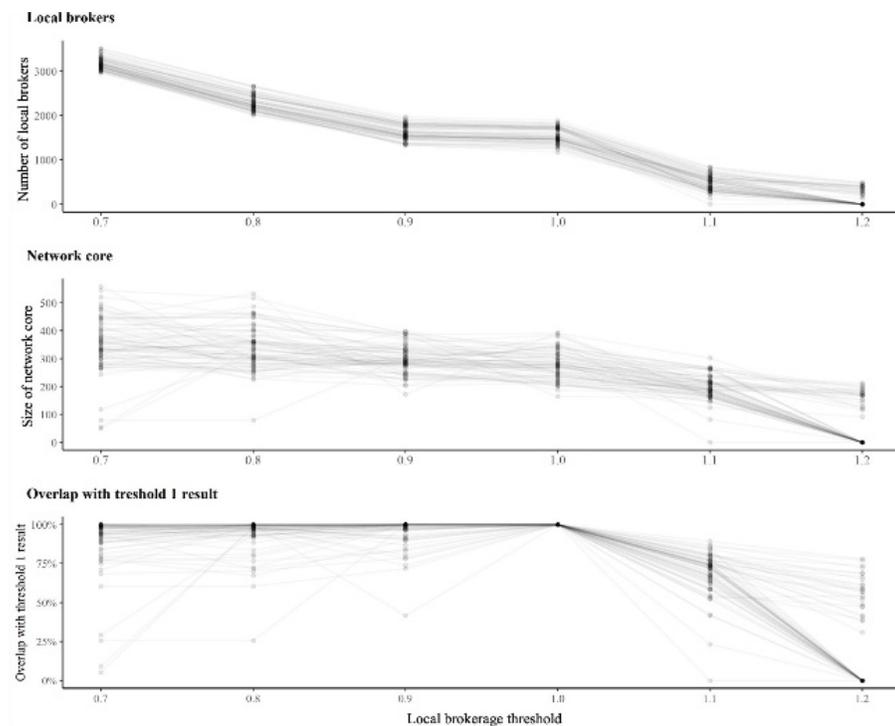


Figure A3. Consequences of adjustment to the local brokerage threshold.

Note: The figure plots the consequences of varying the threshold in the local brokerage pruning for each month in the period 2010 to 2015. Each point represents a month and the lines connect monthly points at different threshold value along the x-axis. Here we let the exclusion threshold for local brokerage increase from 0.7 to 1.2 in steps of 0.1, and report the number of local brokers (panel a) and the network core (panel b) as well as the percentage overlap between the network core identified via the chosen threshold vs. the adjusted threshold (panel c).

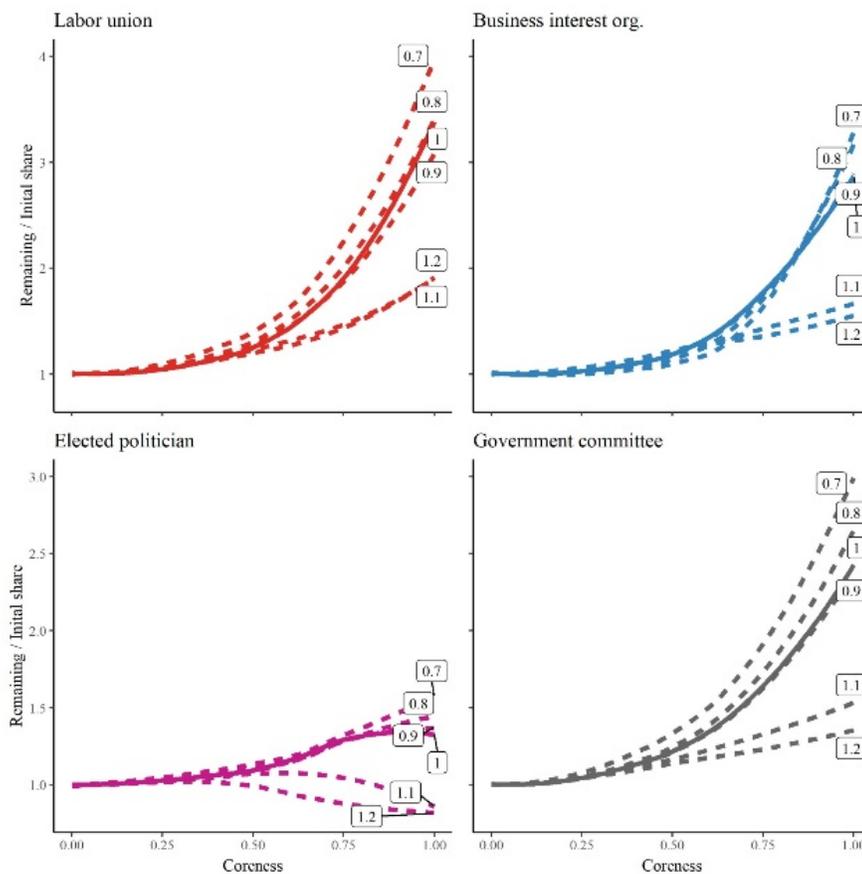


Figure A4. The consequences of the local brokerage threshold on the concentration of political directors in the network core

Note: The figure plots the consequences of varying the threshold on the relative increase in the share of directors at each value of standardized coreness. Lines represent the average over all 72 months (2010-20015), when we let the local brokerage threshold increase from 0.7 to 1.2 in steps of 0.1. Solid lines represent threshold = 1, used in the reported analysis. Baseline share is the share of political directors among the full set of local brokers under the given threshold.