sociological science

Supplement to:

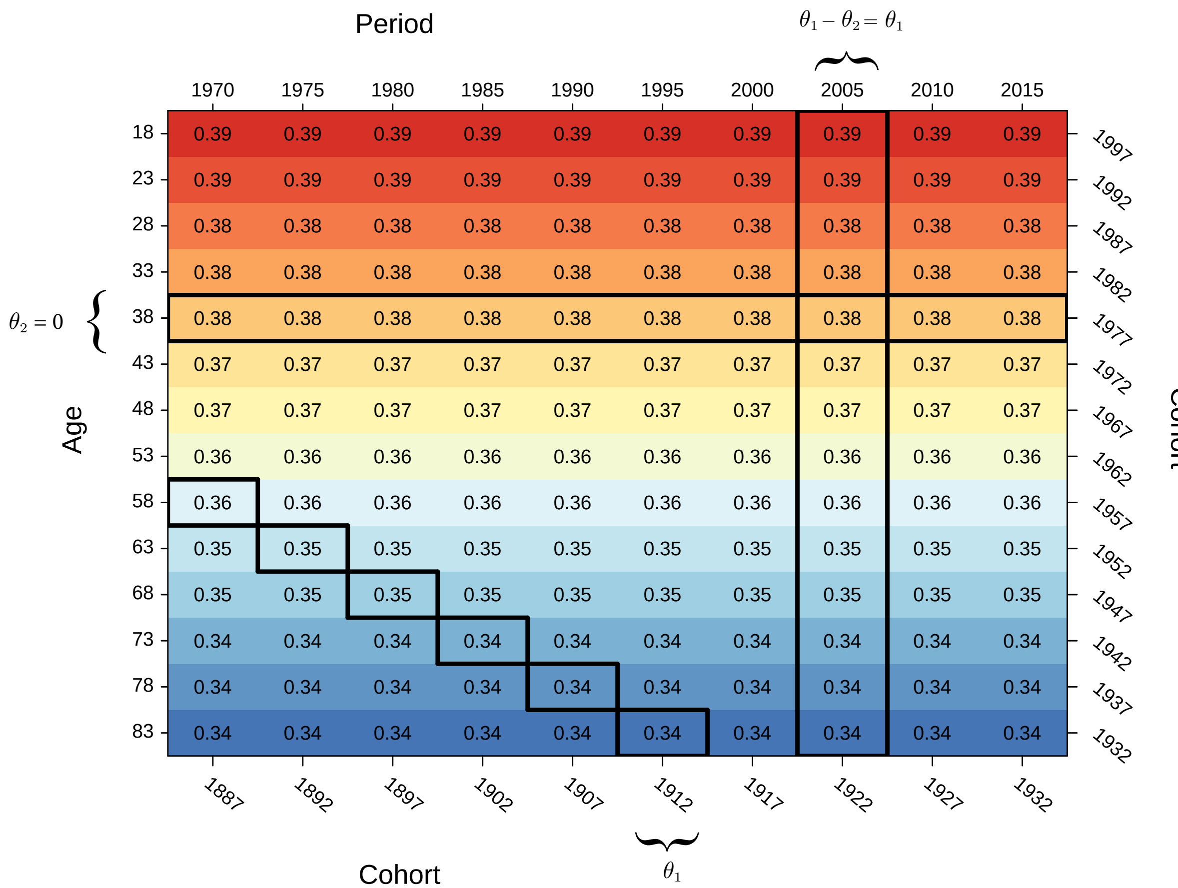
Fosse, Ethan. 2023. "Dissecting the Lexis Table: Summarizing Population-Level Temporal Variability with Age–Period–Cohort Data." Sociological Science 10: 150-196.

Appendix A: 2D Heat Maps of Diachronic Age & Cohort Slopes

2D Heat Map: Diachronic Age Slope

Figure 1 displays a 2D heat map of predicted probabilities for social trust based on the diachronic age slope, or $\theta_1 = \alpha + \pi$, with highlighted diagonal, row, and column sections. The highlighted diagonal is a record of the change in the predicted probability of trust for the cohort with a midpoint birth year of 1912, which is observed to decline from 0.36 among those individuals aged 58-62 in 1979-1974 to 0.34 among those aged 83+ in 1995-1996. This reflects the fact that the diachronic age slope, which summarizes life-cycle change or intra-cohort change, is negative (Log-odds ratio: -0.192; $p = 0.001$). The highlighted row documents the change in the predicted probability of trust for those individuals aged 38-42, which is the same as we compare cohorts of individuals with a midpoint birth year of 1932 and observed in 1970-1974 to those with a midpoint birth year of 1977 and observed in 2015-2019. This is as expected; because we are adjusting for the cohort linear component at its mean, the diachronic cohort slope, or $\theta_2 = \gamma + \pi$, is zero. Finally, the highlighted column documents the difference between age-and-cohort groups for those individuals observed in the 2005-2009 period. However, when the diachronic cohort slope is zero, then the synchronic age slope equals the diachronic age slope. Thus, the decline of the predicted probability of trust from 0.39 to 0.34 in the 2005-2009 cross-section reflects how individuals age through time (i.e., across periods) rather than static age-cohort comparisons. Accordingly, one can just compare across the rows (i.e., age groups) of Figure 1 to summarize how people age through time. Equivalently, one can display a line graph with age as the single axis of variation as in Figure 4(a) in the main text.

Figure 1: 2D Lexis Heat Map of Diachronic Age Slope

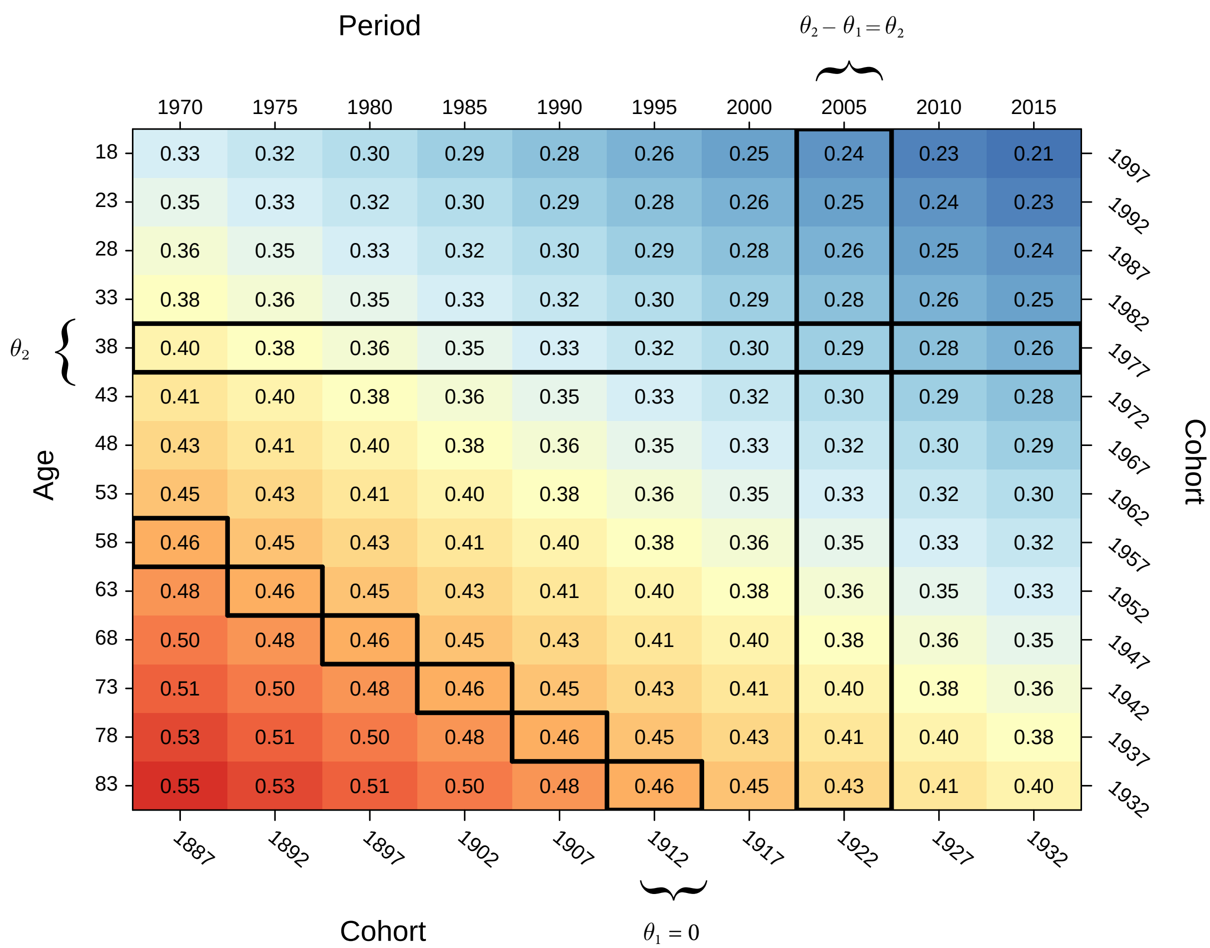


Notes: Figure displays a 2D heat map of predicted probabilities of social trust for all observed combinations of age, period, and cohort. $\theta_1 = \alpha + \pi$ and $\theta_2 = \gamma + \pi$. Calculations based on the diachronic age slope, or θ_1, for all levels of $i = 1, \ldots, I$. Estimates are derived from a diachronic L-APC logistic regression model. Outcome is social trust, coded so that 1 = "can trust" while 0 = "can't be too careful" or "depends." Results adjusted using sampling weights.

2D Heat Map: Diachronic Cohort Slope

Figure 2 displays a 2D heat map of predicted probabilities for social trust based on the diachronic cohort slope, or $\theta_2 = \gamma + \pi$, again with highlighted diagonal, row, and column sections. As with Figure 1, the highlighted diagonal is a record of the change in the predicted probability of trust for the cohort with a midpoint birth year of 1912, which is now the same as we compare those individuals aged 58-62 in 1979-1974 to those aged 83+ in 1995-1996. The reason is that we are adjusting for the age linear component at its mean such that the diachronic age slope, or $\theta_1 = \alpha + \pi$, is zero. The highlighted row documents the change in the predicted probability of trust for those individuals aged 38-42, which declines from 0.40 for those with a midpoint birth year of 1932 and observed in 1970-1974 to 0.26 for those with a midpoint birth year of 1977 and observed in 2015-2019. This reflects the large, negative diachronic cohort slope, which summarizes social change or differences across cohorts through time (Log-odds ratio: -0.677; $p < 0.001$). Lastly, the highlighted column summarizes the difference between age-and-cohort groups for those individuals observed in the 2005-2009 period. Yet, as shown in Table 2 in the main text, if the diachronic age slope is zero, then the synchronic cohort slope equals the diachronic cohort slope. Accordingly, the decline of the predicted probability of trust from 0.43 to 0.24 in the 2005-2009 cross-section reflects how cohorts differ through time (i.e., across periods) rather than static age-cohort comparisons. The consequence is that one can just compare across the diagonals (i.e., cohort groups) of Figure 2 to summarize how cohorts differ through time or, equivalently, one can display a line graph with cohort as the single axis of variation as in Figure 8(a) in the main text.

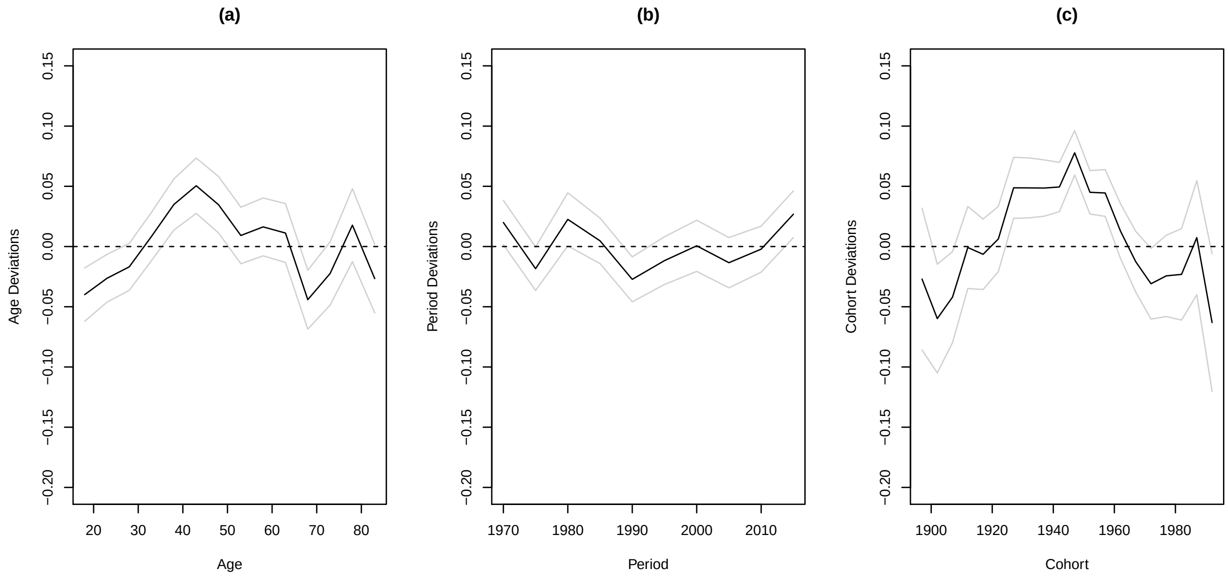
Figure 2: 2D Lexis Heat Map of Diachronic Cohort Slope



Notes: Figure displays a 2D heat map of predicted probabilities of social trust for all observed combinations of age, period, and cohort. Calculations based on the diachronic cohort slope, or $\theta_2 = \gamma + \pi$, for all levels of $k = 1, \ldots, K$. Estimates are derived from a Diachronic L-APC logistic regression model. Outcome is social trust, coded so that 1 = "can trust" while 0 = "can't be too careful" or "depends." Results adjusted using sampling weights.

Appendix B: Nonlinearities for Social Trust

Figure 3: Age, Period, & Cohort Nonlinearities



Notes: Panels **(a)**, **(b)**, and **(c)** display the age, period, and cohort nonlinearities, respectively. All estimates are based on a diachronic L-APC logistic regression model. Upper and lower bounds denote 95% confidence intervals. Outcome is social trust, coded so that 1 = "can trust" while 0 = "can't be too careful" or "depends." Results adjusted using sampling weights.

Appendix C: Adjusted Age, Period, & Cohort Marginal Curves

In analyzing APC data, researchers often present marginal age, period, or cohort curves. In the context of the L-APC model, we can define age, period, and cohort curves that are marginal with respect to the linear components. Because these curves still adjust for all three nonlinearities, we will refer to these as *adjusted marginal curves*. (This is to distinguish these curves from those that do not include all three sets of nonlinearities in the model. Marginal curves that adjust for neither the linear nor nonlinear components of the remaining two variables are beyond the scope of this article.) Interestingly, the slopes of these curves are equal to weighted sums of underlying diachronic and synchronic slopes. In fact, as a measure of overall change, the adjusted marginal period curve is preferred over that of age and cohort because it is a weighted sum of the diachronic age and cohort slopes. In other words, the adjusted marginal period curve is inherently diachronic, reflecting a dynamic, composite summary. By contrast, the adjusted marginal age and cohort curves are mixtures of diachronic and synchronic slopes, conflating dynamic changes with static differences.

To illustrate these relationships, note that we can define three different descriptive models depending on the linear component that is included. First, the *marginal age L-APC model*, which drops the period and cohort linear components, is as follows:

$$Y_{ijk} = \mu + \alpha_M(i - i^*) + \widetilde{\alpha}_i + \widetilde{\pi}_j + \widetilde{\gamma}_k + \epsilon_{ijk}, \quad (1)$$

where α_M is the *adjusted marginal age slope* and $\alpha_M(i - i^*) + \widetilde{\alpha}_i$ is the *adjusted marginal age curve*. Second, the *marginal period L-APC model* is given by:

$$Y_{ijk} = \mu + \pi_M(j - j^*) + \widetilde{\alpha}_i + \widetilde{\pi}_j + \widetilde{\gamma}_k + \epsilon_{ijk} \quad (2)$$

where π_M is the *adjusted marginal period slope* and $\pi_M(j - j^*) + \widetilde{\pi}_j$ is the *adjusted marginal period curve*. Finally, the *marginal cohort L-APC model* is given by:

$$Y_{ijk} = \mu + \gamma_M(k - k^*) + \widetilde{\alpha}_i + \widetilde{\pi}_j + \widetilde{\gamma}_k + \epsilon_{ijk} \quad (3)$$

where γ_M is the *adjusted marginal cohort slope* and $\gamma_M(i - i^*) + \widetilde{\gamma}_k$ is the *adjusted marginal cohort curve*.

As mentioned previously, the slopes of the adjusted marginal age, period, and cohort curves can be interpreted as weighted combinations of underlying diachronic and synchronic slopes. First, the adjusted marginal age curve is equal to the following:

$$\alpha_M(i - i^*) + \widetilde{\alpha}_i = \Big(\theta_1\beta_{(j,i)} + (\theta_2 - \theta_1)\beta_{(k,i)}\Big)(i - i^*) + \widetilde{\alpha}_i \text{ for } i = 1, \ldots, I, \quad (4)$$

where $\beta_{(j,i)}$ is the slope of the period linear component on the age linear component conditional on the age, period, and cohort nonlinearities while $\beta_{(k,i)}$ is the slope of the cohort linear component on the age linear component again conditional on the age, period, and cohort nonlinearities. Because the conditional slope of the cohort linear component on the age linear component, or $\beta_{(k,i)}$, is negative, one can write $(\theta_2 - \theta_1)(-\beta_{(k,i)}) = (\theta_1 - \theta_2)\beta_{(k,i)}$ in Equation 4. Accordingly, the slope of the adjusted marginal age curve is equal to a weighted sum of the diachronic and synchronic age slopes.

Second, the adjusted marginal cohort curve is equivalent to:

$$\gamma_M(k - k^*) + \widetilde{\gamma}_k = \Big((\theta_1 - \theta_2)\beta_{(i,k)} + \theta_2\beta_{(j,k)}\Big)(k - k^*) + \widetilde{\gamma}_k \ \text{ for } \ k = 1, \ldots, K, \quad (5)$$

where $\beta_{(i,k)}$ is the slope of the age linear component on the cohort linear component conditional on the age, period, and cohort nonlinearities while $\beta_{(j,k)}$ is the slope of the period linear component on the cohort linear component again conditional on the age, period, and cohort nonlinearities. Similar to the adjusted marginal age curve, because the conditional slope of the age linear component on the cohort linear component, or $\beta_{(i,k)}$, is negative, one can write $(\theta_1 - \theta_2)(-\beta_{(i,k)}) = (\theta_2 - \theta_1)\beta_{(i,k)}$ in Equation 5. Thus, the slope of the adjusted marginal cohort curve is equal to a weighted sum of the synchronic and diachronic cohort slopes.

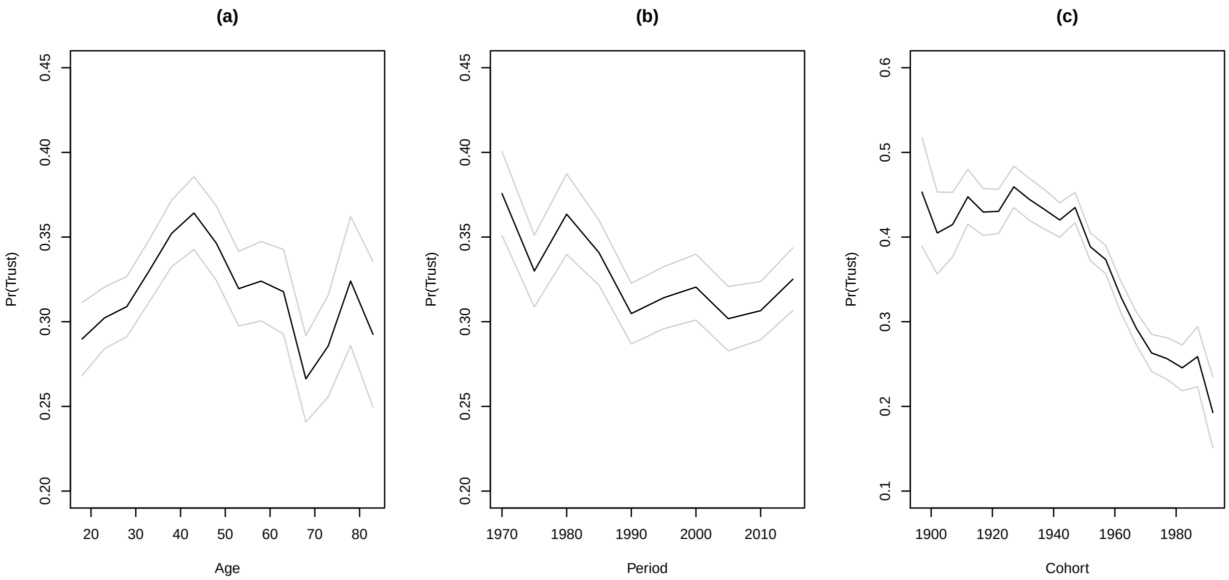
Finally, the adjusted marginal period curve is equal to:

$$\pi_M(j - j^*) + \widetilde{\pi}_j = \Big(\theta_1\beta_{(i,j)} + \theta_2\beta_{(k,j)}\Big)(j - j^*) + \widetilde{\pi}_j \ \text{ for } \ j = 1, \ldots, J, \quad (6)$$

where $\beta_{(i,j)}$ is the slope of the age linear component on the period linear component conditional on the age, period, and cohort nonlinearities while $\beta_{(k,j)}$ is the slope of the cohort linear component on the period linear component again conditional on the age, period, and cohort nonlinearities. Thus, the slope of the adjusted marginal period curve is a weighted sum of the diachronic age and cohort slopes. It is for this reason that the adjusted marginal period curve is preferred over the adjusted marginal age and cohort curves. Unlike those for age and cohort, the adjusted marginal period curve incorporates no static components; as such, it can be interpreted as a composite summary of overall change in an APC data set.

For a fixed number of age groups, the weights for the slopes of the adjusted marginal curves will vary depending on the number of period groups (and, accordingly, cohort groups) in the data. First, if there is a single period then the slope of the adjusted marginal age curve will equal the synchronic age slope. This is because, with a single period, $\beta_{(i,j)}$ is undefined and $\beta_{(k,i)} = -1$, so the slope is$(-1)(\theta_2 - \theta_1) = \theta_1 - \theta_2$. As the number of periods grows (for a given number of age groups), the diachronic cohort slope will tend to dominate the slope of the adjusted marginal age curve. Specifically, the slope of the adjusted marginal age curve will tend to equal the sum of the diachronic age slope and $\beta_{(k,i)}\theta_2$, where $\beta_{(k,i)}$ is an increasingly large, negative weight. Second, if there is just a single time period then the adjusted marginal period curve is undefined because the weights $\beta_{(i,j)}$ and $\beta_{(k,j)}$ will be undefined. This just reflects the fact that, like the diachronic age and cohort slopes, the adjusted marginal period curve is an inherently diachronic measure. With an increasing number of periods (for a fixed number of age groups), the slope of the adjusted marginal period curve will tend to converge on the diachronic cohort slope. This is because the weight $\beta_{(i,j)}$ will approach zero while $\beta_{(k,j)}$ will approach one. Finally, if there is a single period then the slope of the adjusted marginal

Figure 4: Age, Period, and Cohort Marginal Curves



Notes: Panel **(a)** displays predicted probabilities of social trust from the adjusted marginal age curve, or $\left(\theta_1\beta_{(j,i)} + (\theta_2 - \theta_1)\beta_{(k,i)}\right)(i - i^*) + \tilde{\alpha}_i$ for all levels $i = 1, \ldots, I$. Panel **(b)** displays predicted probabilities of social trust from the adjusted marginal period curve, or $\left(\theta_1\beta_{(i,j)} + \theta_2\beta_{(k,j)}\right)(j - j^*) + \tilde{\pi}_j$ for all levels $j = 1, \ldots, J$. Panel **(c)** displays predicted probabilities of social trust from the adjusted marginal cohort curve, or $\left((\theta_1 - \theta_2)\beta_{(i,k)} + \theta_2\beta_{(j,k)}\right)(k - k^*)$ for all levels $k = 1, \ldots, K$. Horizontal axes are expressed in terms of midpoint values of age, period, and cohort groups. Estimates derived from marginal age, period, and cohort L-APC logistic regression models. Outcome is social trust, coded so that 1 = "can trust" while 0 = "can't be too careful" or "depends." Results adjusted using sampling weights.

cohort curve will equal the synchronic cohort slope. This is because, if there is a single period then, like the adjusted marginal age curve, $\beta_{(j,k)}$ is undefined and $\beta_{(i,k)} = -1$, so that $(-1)(\theta_1 - \theta_2) = \theta_2 - \theta_1$. As the number of periods increases (for a fixed number of age groups), the slope of the adjusted marginal cohort curve will tend to converge on the diachronic cohort slope. The reason is that $\beta_{(i,k)}$ will approach zero while $\beta_{(j,k)}$ will approach one.

The adjusted marginal curves for age, period, and cohort for social trust are shown in panels (a), (b), and (c), respectively, of Figure 4. The adjusted marginal age, period, and cohort slopes (in terms of log-odds ratios) are $\alpha_M = 0.080$ ($p = 0.067$), $\pi_M = -0.306$ ($p < 0.001$), and $\gamma_M = -0.563$ ($p < 0.001$). In general, the adjusted marginal age curve suggests a relative increase in social trust, while the adjusted marginal period and cohort curves indicate marked declines. However, as mentioned previously, the adjusted marginal period curve is preferred because it is a weighted sum of the diachronic age and cohort slopes; it is thus an inherently diachronic measure. The weights for the adjusted marginal period curve are $\beta_{(i,j)} = 0.727$ and $\beta_{(k,j)} = 0.273$. Because the diachronic age and cohort slopes are $\theta_1 = -0.171$ and $\theta_2 = -0.668$, respectively, the adjusted marginal period slope is $\theta_1\beta_{(i,j)} + \theta_2\beta_{(k,j)} = (-0.171)(0.727) + (-0.668)(0.273) \approx -0.306$. In other words, the overall observed change across periods is a decline in social trust, which reflects the declines in trust as we track age and cohort groups through time, weighted by the respective positive associations of age and cohort with period.

By contrast, the adjusted age and cohort marginal curves are weighted combinations of diachronic and synchronic slopes. Take, for example, the adjusted marginal age slope of $\alpha_M = 0.080$ ($p = 0.067$). The weights are $\beta_{(j,i)} = 0.626$ and $\beta_{(k,i)} = -0.374$. Given that the the diachronic age slope is 0.626 and the synchronic cohort slope is -0.374, then $\alpha_M = \theta_1 \beta_{(j,i)} + (\theta_2 - \theta_1)\beta_{(k,i)} = (-0.171)(0.626) + (-0.498)(-0.374) \approx 0.080$. That is, the adjusted marginal age slope reflects the decline of trust as we track age groups through time, weighted by the positive association of age with period, as well as a negative synchronic cohort slope, weighted by the negative association of age with cohort. Accordingly, the adjusted marginal age curve is a weighted combination of both dynamic change as well as static differences.