

Supplement to:

Boda, Zsófia. 2018. "Social Influence on Observed Race." Sociological Science 5: 29-57.

Appendix A: Configurations as combinations of SAOM-effects

Appendix A provides more information about how each configuration expressing our hypotheses can be tested in *RSiena*, the implementation of SAOMs. This is necessary since not all of these configurations are directly testable in the model; some should be calculated as a sum of two or more effects. This is because the variables these configurations are built on are dummy variables; therefore, one category (the majority classifications, the majority self-classifications, and the non-friendships) is always the reference category.

We review every hypothesis from this aspect, explaining which SAOM effect, or which combination of multiple effects is able to express the prediction of each hypothesis; providing visual illustration and mathematical explanations in all cases.

The baseline hypothesis (H/baseline)

This hypothesis can be directly expressed by the self-classification of *Alter* effect (Figure 3); that is, if *Alter* is a self-classified minority member (*Alter*'s self-classification=1), *Ego* will be more likely to classify *Alter* as a minority member compared to the reference category (*Alter*'s self-classification= 0, therefore, *Alter* is a self-classified majority member).

Figure A.1: SAOM effect for the baseline hypothesis

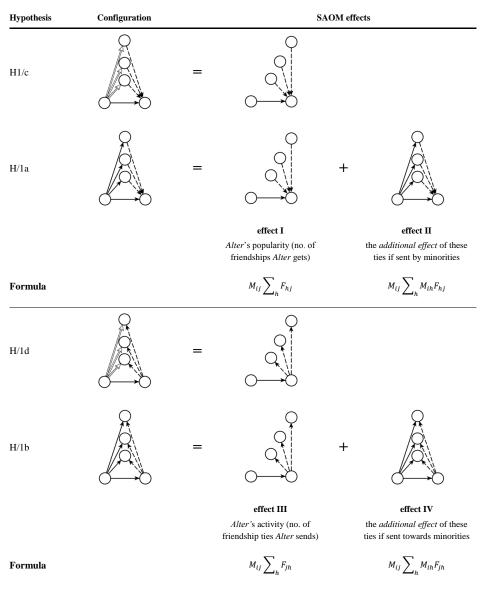
0—	→●
Ego	Alter
m_j	M _{ij}

Alter's self-identification

blank arrow: minority classification; black node: self-classified minority actor; white node: any actor M: minority classification; m: self-classified minority actor; i: Ego; j: Alter (ij: tie from Ego to Alter)

Hypothesis 1

H/1 has four predictions, which cannot be all modeled directly in the analysis. This is because racial classifications are captured by a dummy variable; that is, the presence of a majority classification tie equals the absence of a minority classification tie. Table 1 in the main text described the network substructures we need to test in our model. Table A.1 expresses these configurations using SAOM effects, showing which effect parameters need to be summarized in order to express our hypotheses. Table A.1 shows that some of the configurations we are using are substructures of each other: effect II includes effect I, adding an additional minority nomination from Ego towards those nominating Alter as friends; similarly, effect IV includes effect III. Effects I and III capture the influence of Alter's friends in general; while effects II and IV the influence of Alter's minority-group friends specifically. However, if all effects I-IV are in the model, the general friendship effects of effect I and III should be interpreted by keeping in mind that friendships from/towards minorities are captured by effects II and IV; therefore, the estimated parameters for these effects express the friendship effects from/towards the "reference category", that is, majority students. Therefore, if all four effects are in the model, the parameter values of effect I and effect III directly express H/1c and H/1d. At the same time, the sum of the parameters estimated for effects I and II includes the effects of friendship ties *Alter* gets, with the additional influence of these friendships being sent by minorities, expressing H/1a; similarly, the sum of effect III and IV expresses H/1b.



Reference category: majority – majority nominations Solid black arrow: minority classification; white arrow: majority classification; dashed arrow: friendship tie M: minority classification; F: friendship tie; m: self-classified minority actor i: *Ego*; j: *Alter*; h: other actor (e.g. ij: tie from *Ego* to *Alter*; jh: tie from *Alter* to a third person)

Table A.1: Expressing network configurations with SAOM effects for Hypothesis 1

Here we present a short summary and formal reformulation of the above argument. Including effects I-IV together, *RSiena* estimates the following:

$$\beta_{I}M_{ij}\sum\nolimits_{h}F_{hj}+\beta_{II}M_{ij}\sum\nolimits_{h}M_{ih}F_{hj}+\beta_{III}M_{ij}\sum\nolimits_{h}F_{jh}+\beta_{IV}M_{ij}\sum\nolimits_{h}M_{ih}F_{jh}$$

which can also be expressed as

$$\begin{split} (\beta_{\mathrm{I}}+\beta_{\mathrm{II}}) \mathrm{M}_{ij} \sum_{\mathrm{h}} \mathrm{F}_{\mathrm{h}j} \mathrm{M}_{\mathrm{i}\mathrm{h}} &+ \beta_{\mathrm{I}} \mathrm{M}_{ij} \sum_{\mathrm{h}} \mathrm{F}_{j\mathrm{h}} \left(1-\mathrm{M}_{\mathrm{i}\mathrm{h}}\right) + \\ (\beta_{\mathrm{III}}+\beta_{\mathrm{IV}}) \mathrm{M}_{ij} \sum_{\mathrm{h}} \mathrm{F}_{j\mathrm{h}} \mathrm{M}_{\mathrm{i}\mathrm{h}} &+ \beta_{4} \mathrm{M}_{ij} \sum_{\mathrm{h}} \mathrm{F}_{j\mathrm{h}} \left(1-\mathrm{M}_{\mathrm{i}\mathrm{h}}\right). \end{split}$$

In this case,

$$(\beta_I + \beta_{II})M_{ij}\sum\nolimits_h F_{hj}M_{ih} \,\, \text{expresses} \, \beta_{H/1a}$$

since it is referring to those cases where *Alter's* in-coming friendship ties are sent by those classified as minorities by *Ego*, while

$$\beta_{I}M_{ij}\sum\nolimits_{h}F_{hj}\left(1-M_{ih}\right) expresses \ \beta_{H/1c}$$

as it expresses the effect of incoming friendship ties from individuals classified as majorities (captured by the $1 - M_{ih}$ component). Similarly,

$$(\beta_{III} + \beta_{IV})M_{ij}\sum_{h}F_{jh}M_{ih}$$
 expresses $\beta_{H/1b}$

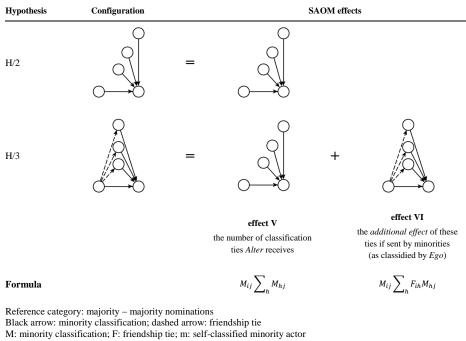
because it captures the effect of the outgoing nominations sent by *Alter* towards those classified as minorities by *Ego*. Finally,

$$\beta_4 M_{ij} \sum_h F_{hj} (1 - M_{ih}) \text{ expresses } \beta_{H/1d}$$

as it equals to the effect of friendships sent towards those classified as majorities by Ego.

Hypotheses 2 and 3

Similarly to the predictions of H/1, H/2 and H/3 cannot be all modeled together directly in the analysis. Table 2 in the main text showed the two configurations we need to test in our model. Table 6 expresses these configurations by either one or the combination of two SAOM effects.



i: Ego; j: Alter; h: other actor (e.g. ij: tie from Ego to Alter; jh: tie from Alter to a third person)

Table A.2: Expressing network configurations with SAOM effects for Hypotheses 2 and 3

The peer influence effect models how much each minority classification Alter gets will

increase the likelihood that Alter will be nominated as a minority member by Ego as well. Here, a

positive parameter value is expected. The friend influence effect models the extra effect of these minority classifications if they are coming from Ego's friends. As friends are expected to be more influential on Ego's classifications than others, here also a positive parameter value is expected. The total influence of Ego's friends can be expressed by the sum of these two parameters.

Estimating the peer influence and the friend influence effects together, the part of the evaluation function estimated by *RSiena* is

$$\beta_{\rm V} M_{ij} \sum_{h} M_{hj} + \beta_{\rm VI} M_{ij} \sum_{h} F_{ih} M_{jh}$$

which can also be expressed as

$$(\beta_{\rm V} + \beta_{\rm VI})M_{ij}\sum_{h}F_{ih}M_{jh} + \beta_{\rm V}M_{ij}\sum_{h}(1 - F_{ih})M_{jh}$$

where the first part expresses the cases when the Roma nominations are sent to *Alter* by *Ego*'s friends (F_{jh}) , therefore

$$(\beta_{\rm V} + \beta_{\rm VI}) M_{ij} \sum_{h} F_{ih} M_{jh}$$
 expresses $\beta_{\rm H/3}$

and the second part expresses the cases when they are sent by other peers, that is, Ego's "non-friends" $(1 - F_{jh})$. Consequently,

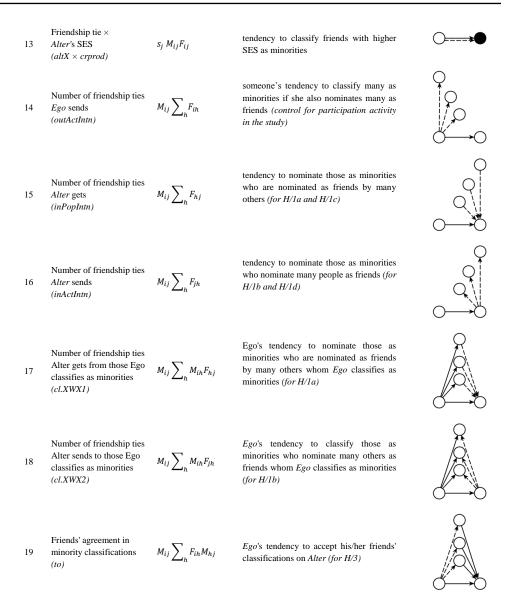
$$\beta_{\rm V} M_{ij} \sum_{h} (1 - F_{ih}) M_{jh}$$
 expresses $\beta_{\rm H/2}$

Appendix B: List of effects estimated in the final model

Dependent variable: Racial classification

Ego nominates Alter as a minority, creating or maintaining the described substructure

	Name (Siena effect name)	Formula	Description	Illustration
	Same-network structural			
1	Outdegree (density)	M _{ij}	"intercept" – a minority classification exists when every parameter is 0 (1=tie, 0=no-tie)	$\bigcirc \longrightarrow \bigcirc$
2	Reciprocity (recip)	$M_{ij}M_{ji}$	<i>Ego</i> 's tendency to reciprocate minority classifications	$\bigcirc \longrightarrow \bigcirc$
3	Number of minority classifications <i>Alter</i> gets (<i>inPop</i>)	$M_{ij}\sum_{h}M_{hj}$	tendency to classify those as minorities who are classified as minorities by many (for H2 and H3)	
4	Number of minority classifications <i>Ego</i> sends (<i>outAct</i>)	$M_{ij}\sum_{h}M_{ih}$	tendency of those classifying many others as minorities to classify Alter this way as well (control for how active Ego is at nominating others as minorities)	
	Covariate effects			
5	<i>Ego</i> 's self-classification (<i>egoX</i>)	$m_i M_{ij}$	tendency of a self-identified minorities to classify more others as minorities	●→─
6	Alter's self-classification (altX)	$m_j M_{ij}$	tendency of a self-identified minorities to be classified as minorities (for H/b)	◯──●
7	Ego's SES (egoX)	$s_i M_{ij}$	tendency of those with higher SES to classify more as minorities (SES control for Effect 5)	●→─
8	Alter's SES (altX)	s _j M _{ij}	tendency of those with higher SES to be classified as minorities (SES control for Effect 6)	$\bigcirc \longrightarrow \blacksquare$
	Mixed-network effects			
9	Friendship tie (crprod)	$M_{ij}F_{ij}$	tendency to classify friends as minorities	○₽○
10	Friendship tie \times Ego's self-classification (egoX \times crprod)	$m_i M_{ij} F_{ij}$	minority individuals' tendency to classify their friends as minorities	••
11	Friendship tie × Alter's self-classification (altX × crprod)	$m_j M_{ij} F_{ij}$	tendency to classify self-identified minority friends as minorities	○▶●
12	Friendship tie × Ego's SES $(egoX \times crprod)$	$s_i M_{ij} F_{ij}$	tendency of those with higher SES to classify their friends as minorities	••



-	endent variable: <i>friendship</i>		· · · · · · · · · · · · · · · · · · ·	
r.go		Formula	Description	Illustration
	Name Same-network structural	i of filula	Description	maduation
20	Outdegree (<i>density</i>)	F _{ij}	"intercept" – a friendship nomination exists when every parameter is 0	○•○
21	Reciprocity (recip)	F _{ij} F _{ji}	(1=tie, 0=no-tie) Ego's tendency to reciprocate friendship nominations (reciprocating is one of the most important explanation of friendships)	○ ₹=== ≥ ○
22	Transitivity (transTrip)	$\sum\nolimits_{h}F_{ij}F_{ih}F_{hj}$	tendency for clustering (befriending friends of friends is one of the most important explanation of friendships)	
23	Number of friendships Ego sends (outAct)	$F_{ij}\sum_{h}F_{ih}$	tendency of those <i>Egos</i> who nominate many others as friends to nominate <i>Alter</i> as well <i>(control for "activeness" of Ego)</i>	○ ○ / ○ / ○ / ○
24	Number of friendships Alter gets (inPop)	$F_{ij}\sum_{h}F_{hj}$	tendency to nominate those as friends who are nominated by many (<i>capturing the</i> "Matthew effect", that is, popularity usually induces more popularity)	
	Covariate effects			
25	<i>Ego</i> 's self-classification (<i>egoX</i>)	$m_i F_{ij}$	tendency of self-identified minorities to send more friendship nominations	●>○
26	<i>Alter</i> 's self-classification (<i>altX</i>)	$m_j F_{ij}$	tendency to nominate more self-identified minorities as friends	○ ►
27	Ego×Alter self-class. (egoXaltX)	$m_i m_j F_{ij}$	tendency of self-identified minorities to nominate more self-identified minorities (racial homophily in friendships)	●>●
28	Ego's SES (egoX)	s _i F _{ij}	tendency of <i>Egos</i> with higher SES to send more friendship nominations (<i>SES control</i> <i>for Effect 25</i>)	●•○
29	Alter's SES (altX)	s _j F _{ij}	tendency to nominate more <i>Alters</i> as friends whose SES is higher (SES control for Effect 26)	○ ►
30	Ego imes Alter SES (egoXaltX)	s _i s _j F _{ij}	tendency of <i>Egos</i> with higher SES to nominate more <i>Alters</i> with higher SES (SES-homphily in friendships)	●>●

31	Ego's Gender (egoX)	$g_i F_{ij}$	tendency of females to send more friendship nominations	●>○
32	Alter's Gender (altX)	$g_j F_{ij}$	tendency to nominate more females as friends	○ ►●
33	Ego imes Alter Gender ($egoXaltX$)	$g_i g_j F_{ij}$	tendency of females to nominate each other more (gender homphily)	●>●
	Mixed-network effects			
34	Minority tie (crprod)	$F_{ij}M_{ij}$	Egos' tendency to send friendship nominations towards those whom they classify as minorities (modeling the opposite direction of Effect 9)	○ ≱○
35	Minority tie × Ego's self-classification (egoX × crprod)	$m_i F_{ij} M_{ij}$	self-identified minority <i>Egos</i> ' tendency to send friendship nominations towards those whom they classify as minorirites (modeling the opposite direction of Effect 10; minority and majority students might befriend differently with others they classify as minorities)	●>○
36	Minority tie \times Ego's SES (egoX \times crprod)	$s_i F_{ij} M_{ij}$	higher-SES <i>Egos'</i> tendency to send friendship nominations towards those whom they classify as minorities (<i>SES</i> <i>control for Effect 35</i>)	●¥○
37	Minority in-degree of Ego (inActIntn)	$F_{ij}\sum_{h}M_{hi}$	tendency of those classified as minorities by many to nominate more as friends (modeling the opposite direction of Effect 16)	↓ ↓ ↓ ↓
38	Minority in-degree of Alter (inPopIntn)	$F_{ij}\sum_{h}M_{hj}$	tendency to nominate those as friends who are classified as minorities by many (modeling the other direction of Effect 15)	0
39	Minority in-degree of Alter \times Ego's self-classification (egoX \times inPopIntn)	$m_i F_{ij} \sum_h M_{hj}$	minority $Egos^{i}$ tendency to nominate those as friends who are classified as minorities by many (capturing the effect that minority students might befriend differently those classified as minorities by many)	••
40	Minority in-degree of Alter × Ego's SES (egoX × inPopIntn)	$s_i F_{ij} \sum_h M_{hj}$	higher-SES <i>Egos</i> ' tendency to nominate those as friends who are classified as minorities by many (<i>SES control for Effect</i> 39)	

solid black arrow: minority classification; dashed black arrow: friendship tie black node: self-classified minority actor /actor with higher SES / female actor; white node: any actor M: minority classification; F: friendship tie; m: self-classified minority actor; i: Ego; j: Alter; h: other actor (e.g. ij: tie from Ego to Alter; jh: tie from Alter to a third person)

Appendix C: Descriptive statistics

Ge	nder		Father's level of education		
Female	Male	Primary or less	Secondary	Tertiary	Missing
223 (62%)	134 (38%)	80 (22%)	232 (65%)	20 (6%)	25 (5%)

Table C.1: Gender and father's level of education

Total s	ample		Classroom level			
Minority	Missing	SD of averages	Min	Max		
96 (27%)	7 (2%)	19.1%	11%	74%		

Table C.2: Racial self-classification

		Density		Jaccard	Jaccard-index	
		Mean	SD	Mean	SD	Proportion
Friendship	Wave 1	0.20	0.02	0.34	0.05	0.05
	Wave 2	0.19	0.03	0.34	0.05	0.14
Racial classification	Wave 1	0.15	0.13			0.06
	Wave 2	0.21	0.20	0.36	0.14	0.10

Table C.3: Friendship networks and racial classification networks

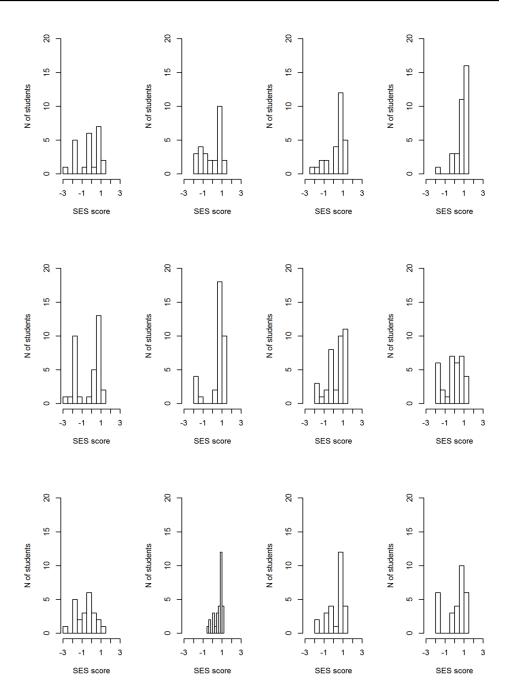


Figure C.1: SES distribution across classrooms

Appendix D: Details of the procedure

We estimated the parameters using the *sienaBayes* function, which is part of the R-package *RSienaTest* (Ripley et al., 2016). For the estimation, we used a Bayesian MCMC procedure (Gelman et al., 2013). Throughout the process, we followed the guidelines of Koskinen and Snijders (2017).

The model had 64 parameters for the 12 groups: 24 rate parameters, and 40 global parameters. Rate parameters were different for each group: we thus had 12 rate parameters for the perception network, and 12 for the friendship network. Of the global parameters, we specified fixed and random effects. A fixed effect is the same for all groups. For a random effect, the parameter varies between groups following a multivariate normal distribution. In our model, 28 global effects were fixed, and 12 were random. To obtain a good power, all parameters playing a role in hypothesis tests were set to fixed. The table in Appendix E gives the list of all parameters and whether they were random or fixed.

For the Bayesian analysis, we specified a prior distribution for the randomly varying parameters. We chose for a weakly informative prior. The priors for rate parameters were datadependent. For the network parameters, priors were given a prior mean 0, with the exception of the outdegre parameter, where the prior mean was negative, and the reciprocity and transitivity parameters, where the prior means were positive, based on existing knowledge of social network models. Prior standard deviations were chosen in the range of 0.07 to 0.6, also in line with experience with this type of models.

For assessing convergence, we followed the guidelines of Gelman et al. (2013). Four separate MCMC runs were made, each consisting of 1000 samples after thinning. Graphical assessment of the trace plots showed good convergence. Then we used the R package *rstan* to calculate the convergence indicator R-hat. For global parameters, values lower than 1.1 are considered good; in

our case, this was less than 1.08 for every parameter (Gelman et al., 2013). Therefore, convergence of the MCMC process was good.

Finally, we conducted additional statistical tests to calculate parameter values for the hypotheses which were not directly tested in the model. For this, we calculated Mahalanobis distances of the elements of the posterior sample from the posterior mean for linear combinations of multiple effects, where the p-value is the relative frequency that these are greater than the distance between the tested value and the posterior mean.

Appendix E: Results

Dependent variable: racial classification

Ego nominates Alter as a minority, creating or maintaining the described substructure

Name	Estimate	S.D.	Credible interval (95%)		Posterior probability	Varying
Same-network structural						
1 Outdegree	- 3.716	0.415	- 4.532	- 2.905	0.00	+
2 Reciprocity	- 0.227	0.129	- 0.473	0.0212	0.04	_
3 Number of minority classifications Alter gets	0.129	0.011	0.108	0.150	1.00	_
4 Number of minority classifications Ego sends	0.066	0.101	0.049	0.085	1.00	_
Covariate effects						
5 Ego's self-classification	0.176	0.117	- 0.060	0.394	0.93	_
6 Alter's self-classification	0.857	0.104	0.663	1.060	1.00	_
7 Ego's SES	- 0.044	0.048	- 0.143	0.046	0.17	_
8 Alter's SES	- 0.019	0.039	- 0.096	0.057	0.32	_
Mixed-network effects						
9 Friendship tie	0.102	0.274	- 0.461	0.628	0.66	_
10 Friendship tie $\times Ego$'s self-classification	0.507	0.239	0.052	0.978	0.99	_
11 Friendship tie × Alter's self-classification	0.585	0.232	- 0.390	0.527	0.59	_
12 Friendship tie $\times Ego$'s SES	0.098	0.132	- 0.151	0.371	0.77	_
13 Friendship tie × Alter's SES	- 0.114	0.110	- 0.335	0.098	0.15	_
14 Number of friendship ties Ego sends	- 0.044	0.019	- 0.084	- 0.008	0.01	_
15 Number of friendship ties Alter gets	- 0.084	0.029	- 0.143	- 0.029	0.00	_
16 Number of friendship ties Alter sends	0.041	0.016	0.011	0.074	1.00	-
17 Number of friendship ties Alter gets from those Ego classifies as minorities	0.476	0.106	0.280	0.696	1.00	_
18 Number of friendship ties Alter sends to those Ego classifies as minorities	- 0.082	0.083	-0.248	0.073	0.16	_
19 Friends' agreement in minority classifications	0.053	0.343	-0.030	0.142	0.90	-

Dependent	variable:	friendship	,

Ego nominates Alter as a friend, creating or maintaining the described substructure

Name	Estimate	S.D.	Credible interval (95%)		Posterior probability	Varying
Same-network structural						
20 Out-degree	- 1.760	0.359	- 2.469	- 0.057	0.00	+
21 Reciprocity	1.157	0.102	0.961	1.359	1.00	+
22 Transitive triplets	0.247	0.067	0.113	0.380	1.00	+
23 Number of friendships Alter gets	- 0.075	0.067	- 0.206	0.055	0.13	+
24 Number of friendships Ego sends	0.000	0.061	- 0.119	0.120	0.50	+
Covariate effects						
25 Ego's self-classification	0.046	0.140	-0.231	0.319	0.63	+
26 Alter's self-classification	0.363	0.160	0.065	0.690	0.99	+
27 Ego \times Alter self-classification	0.607	0.170	-0.105	0.538	0.88	-
28 Ego's SES	- 0.029	0.085	- 0.195	0.140	0.36	+
29 Alter's SES	- 0.022	0.083	- 0.189	0.140	0.39	+
$30 Ego \times Alter SES$	0.013	0.072	- 0.130	0.157	0.58	+
31 Ego's Gender	- 0.143	0.082	- 0.306	0.020	0.04	-
32 Alter's Gender	- 0.050	0.089	- 0.229	0.122	0.28	-
33 Ego×Alter Gender	0.213	0.078	0.064	0.367	1.00	-
Mixed-network effects						
34 Minority tie	- 0.045	0.245	- 0.523	0.114	0.41	-
35 Minority tie $\times Ego$'s self-classification	0.007	0.261	- 0.507	0.506	0.52	_
36 Minority tie $\times Ego$'s SES	0.144	0.140	- 0.130	0.421	0.85	-
37 Minority in-degree of Ego	- 0.007	0.006	- 0.019	0.006	0.13	-
38 Minority in-degree of Alter	- 0.071	0.062	- 0.197	0.419	0.12	+
39 Minority in-degree of <i>Alter</i> × <i>Ego</i> 's self- classification	0.021	0.015	- 0.007	0.054	0.92	-
40 Minority in-degree of <i>Alter</i> × <i>Ego</i> 's SES	- 0.009	0.007	- 0.022	0.004	0.08	-

References

Gelman, A., Carlin, J. B., Stern, H. S., Dunson, D. B., Vehtari, A. and Rubin, D. B. 2014. *Bayesian data analysis*. Boca Raton, FL: CRC press.

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