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Supplement to:

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Online Supplement: Model Presentation and Interpretation

In this supplement, we present the Endogenous Switching Regression Model (ESRM) and discuss model selection and the main empirical results.

The Endogenous Switching Regression Model

The ESRM considers two outcomes jointly: (1) the likelihood that high-/low-SES parents belong to the high rather than the low cultural capital input state and (2) the effect of being in the high/low cultural capital input state on children's educational attainment (Heckman 1990; Maddala 1983; Mare and Winship 1988). Let z be a binary indicator of the level of cultural capital inputs that parents provide to children. Parents can be located in one of two states: a low- ($z=0$) or a high-input ($z=1$) state. Let y_1 and y_2 be the educational attainment of children whose parents are in the high- and low-input state, respectively. Finally, let **x** and **e** be vectors of observed variables, with the **x** vector including the summary scale of parental SES (and the control variables) and the **e** vector including the two variables which act as exclusion restrictions (described in the main text). Under these definitions, we write the ESRM

$$y_1 = \mathbf{x}\boldsymbol{\beta}_\mathbf{1} + \varepsilon_1 \quad \text{if} \quad z = 1 \tag{1}$$

$$y_2 = \mathbf{x}\boldsymbol{\beta}_\mathbf{2} + \varepsilon_2 \quad \text{if} \quad z = 0 \tag{2}$$

$$z = I\left(\mathbf{x}\boldsymbol{\pi} + \mathbf{e}\boldsymbol{\gamma} + \varepsilon_3 > 0\right), \tag{3}$$

Equations 1 and 2 capture that factors which explain children's educational attainment may be different in the high- and low-input state, respectively. $I(.)$ is an indicator function which takes the values 0 and 1 and which describes the likelihood of belonging to the high-input (as opposed to the low-input) state as a function of the **x** and **e** variables. Equation 3 thus captures the selection process that sorts parents into either a group that provides high cultural capital inputs or a group that provides low inputs. We model Equation 3 as a probit model and furthermore assume that the error terms in all three equations $(\varepsilon_1, \varepsilon_2, \varepsilon_3)$ follow a trivariate normal distribution with variance-covariance matrix

$$\Sigma = \begin{pmatrix} \sigma_1^2 & . & . \\ . & \sigma_2^2 & . \\ \sigma_{13} & \sigma_{23} & 1 \end{pmatrix}. \qquad (4)$$

We estimate the three equations in the ESRM model jointly by means of full information maximum likelihood, and because our analyses involve predictions based on PCA (i.e., variables that we construct) we use bootstrapped standard errors. The covariance parameters σ_{13} and σ_{23} in (4) are informative about selection into the two cultural capital input states (high/low) based on variables that we do not observe in our data. And even though we have no substantive interest in the unobserved factors that drive selection, the inclusion of the covariance parameters means that the ESRM controls for both observed and unobserved factors that affect the likelihood that parents belong to the high-/low-input state and children's educational attainment in each state.

The variables in the $\mathbf{e}$ vector act as so-called *exclusion restrictions*, and they are assumed to affect parents' cultural capital inputs but, conditional on the other explanatory variables in the model (those in the $\mathbf{x}$ vector), are assumed not to have any direct effect on children's educational attainment (other than that going through parents' cultural capital inputs). As we explain in the main text, we use two exclusion restrictions: grandparents' highest years of completed schooling and the mother's self-report of how many years of schooling she expected to complete (measured between age 14-21).

Model Predictions

From the model defined in Equations 1-4 children's expected years of schooling are given by

$$E\left(y_2 \mid z=0,\mathbf{x}\right)=\mathbf{x}\boldsymbol{\beta}_2-\sigma_2\rho_2\frac{\phi\left(\mathbf{x}\boldsymbol{\pi}+\mathbf{e}\boldsymbol{\gamma}\right)}{1-\Phi\left(\mathbf{x}\boldsymbol{\pi}+\mathbf{e}\boldsymbol{\gamma}\right)} \quad \text{(A)}$$

$$E\left(y_1 \mid z=0,\mathbf{x}\right)=\mathbf{x}\boldsymbol{\beta}_1-\sigma_1\rho_1\frac{\phi\left(\mathbf{x}\boldsymbol{\pi}+\mathbf{e}\boldsymbol{\gamma}\right)}{1-\Phi\left(\mathbf{x}\boldsymbol{\pi}+\mathbf{e}\boldsymbol{\gamma}\right)} \quad \text{(B)}$$

$$E\left(y_2 \mid z=1,\mathbf{x}\right)=\mathbf{x}\boldsymbol{\beta}_2+\sigma_2\rho_2\frac{\phi\left(\mathbf{x}\boldsymbol{\pi}+\mathbf{e}\boldsymbol{\gamma}\right)}{\Phi\left(\mathbf{x}\boldsymbol{\pi}+\mathbf{e}\boldsymbol{\gamma}\right)} \quad \text{(C)}$$

$$E\left(y_1 \mid z=1,\mathbf{x}\right)=\mathbf{x}\boldsymbol{\beta}_1+\sigma_1\rho_1\frac{\phi\left(\mathbf{x}\boldsymbol{\pi}+\mathbf{e}\boldsymbol{\gamma}\right)}{\Phi\left(\mathbf{x}\boldsymbol{\pi}+\mathbf{e}\boldsymbol{\gamma}\right)} \quad \text{(D)}$$

where $\rho_1=\frac{\sigma_{13}}{\sigma_1\sigma_3}$ and $\rho_2=\frac{\sigma_{23}}{\sigma_2\sigma_3}$, and $\phi(.)$ and $\Phi(.)$ are the p.d.f. and c.d.f. of the standard normal distribution. Equations (A) through (D) correspond to the predictions shown in Table 3.

Counterfactual Scenarios

We define the three counterfactual scenarios on the basis of model predictions. Let *S* denote parents' socioeconomic status (SES). *S* takes on values *j = 1,2* to denote low and high SES. For children whose parents belong to SES group *j*, let $\bar{Y}_j^{0|0}$ and $\bar{Y}_j^{1|1}$ denote the average educational attainment of children exposed to the low-input state and the high-input state, respectively. Let $\bar{Y}_j^{1|0}$ denote the counterfactual educational attainment of children exposed to the low-input state if they had instead been exposed to the high-input state. Let $\bar{Y}_j^{0|1}$ denote the opposite counterfactual attainment. Finally, let $w_j=\Pr\left(z=1 \mid S=j\right)$ denote the share of high-input parents in SES group *j*. The *observed socioeconomic gradient* between groups *j = 1* and *j = 2* with regard to children's educational attainment is then

$$\bar{Y}_2^{0|0}(1-w_2)+\bar{Y}_2^{1|1}w_2$$
$$-\left[\bar{Y}_1^{0|0}(1-w_1)+\bar{Y}_1^{1|1}w_1\right]$$

To define the three counterfactual scenarios that we analyze, let $q=[0,1]$ be a probability that shifts the relative weight of potential outcomes. We present the three scenarios below.

Scenario (A): Equalization from below

$$CD_A=\bar{Y}_2^{0|0}(1-w_2)+\bar{Y}_2^{1|1}w_2-\left(\left[\bar{Y}_1^{0|0}(1-q)+\bar{Y}_1^{1|0}q\right](1-w_1)+\bar{Y}_1^{1|1}w_1\right)$$

Derivative of CD_A with respect to *q*:

$$\frac{dCD_A}{dq}=-(1-w_1)\left(\bar{Y}_1^{1|0}-\bar{Y}_1^{0|0}\right)$$

Scenario (B): Equalization from above

$$CD_B=\bar{Y}_2^{0|0}(1-w_2)+\left[\bar{Y}_2^{0|1}q+\bar{Y}_2^{1|1}(1-q)\right]w_2-\left(\bar{Y}_1^{0|0}(1-w_1)+\bar{Y}_1^{1|1}w_1\right)$$

Derivative of CD_B with respect to *q*:

$$\frac{dCD_B}{dq}=-w_2\left(\bar{Y}_2^{1|1}-\bar{Y}_2^{0|1}\right)$$

Scenario (C): Equalization by universal intervention

$$CD_C=\left[\bar{Y}_2^{0|0}(1-q)+\bar{Y}_2^{1|0}q\right](1-w_2)+\bar{Y}_2^{1|1}w_2-\left(\left[\bar{Y}_1^{0|0}(1-q)+\bar{Y}_1^{1|0}q\right](1-w_1)+\bar{Y}_1^{1|1}w_1\right)$$

Derivative of CD_C with respect to *q*:

$$\frac{dCD_C}{dq}=(1-w_2)\left(\bar{Y}_2^{1|0}-\bar{Y}_2^{0|0}\right)-(1-w_1)\left(\bar{Y}_1^{1|0}-\bar{Y}_1^{0|0}\right)$$

Scenario (A) captures the socioeconomic gradient that would be observed if the level of cultural capital inputs among low-input/low-SES parents increased by *q*. Scenario (B) captures the gradient that would be observed if the level of cultural capital inputs among high-input/high-SES parents decreased by *q*. Scenario (C) captures the gradient that would be observed if all low-input parents in the population (i.e., both low- and high-SES parents) increased their input level by *q*. The derivatives reported for each scenario describe the impact on the *socioeconomic gradient* of a marginal change in *q*.

Model Selection

Table S1 summarizes model fit statistics for five empirical specifications of the ESRM using the NLSY79 and CYA data. All specifications include the same observed explanatory variables but impose different restrictions on the covariance (correlation) structure of the error terms in the ESRM. These restrictions capture different approaches to measuring the effect of unobserved factors on parents' cultural capital inputs and children's educational attainment. The table includes three measures of model fit: Deviance, the Akaike Information Criterion (AIC), and the Bayesian Information Criterion (BIC). In all cases, lower values imply better fit. As explained in the main text, we estimate the ESRM separately for low- and high-SES parents. In the low-SES group, Model II has the better fit to the data. This model restricts the covariance parameters to be equal. The estimated correlation in this model is -0.59 (Table S2), suggesting that selection into the high investment state is positive, whereas selection into the low investment state is negative. In the high-SES group, Model III has the better fit to the data. In this model, there is selection into the high investment state only. The correlation is -0.59, indicating that selection into this investment state is positive.

Main Results from ESRMs

Table S2 summarizes results for ESRMs estimated among low-/high-SES parents and includes two sets of results: (1) results from regressions of children's educational attainment within each cultural capital input state (low/high; cf. Equations 1 and 2) and (2) results from the selection equation predicting the probability of belonging to the high-input state rather than the low-input state (cf. Equation 3).

Panel A in Table S2 shows empirical estimates for the preferred ESRM among low-SES parents (model II, see Table S1). Results from the selection equation show that the likelihood of belonging to the high (rather than the low) cultural capital input state is higher if parents have higher SES (there is variation in SES – measured via the continuous SES variable – even within the low-SES group). Consequently, parents with high SES are more likely to belong to the group that provides high cultural capital inputs than are parents with low SES. The likelihood of belonging to the high-input group is also higher is the mother is married and is lower if parents have more children or are Black or Hispanic (compared to white). Finally, we find that, as expected, the two exclusion restrictions: grandparents' education and mother's educational expectations in adolescence, are positively correlated with the likelihood of belonging to the high-input state. Panel A also shows results from regressions of children's educational attainment within each cultural capital input state. These results show that children's educational attainment depends on family background characteristics to some extent. Panel B in Table S2 summarizes results for the preferred ESRM in the high-SES group (model III, see Table S1). The substantive results for the selection and outcome models are very similar to those reported in the low-SES group.

Table S1: Model Selection by Parental SES

Model:		Parental SES: Low Deviance	DF	AIC	BIC	Parental SES: High Deviance	DF	AIC	BIC
I	Unconstrained	7956.9	33	8022.9	8191.3	8043.2	33	8109.2	8276.6
II	$\sigma_{13} = \sigma_{23}\ \sigma_{13}\ \sigma_{23} = \sigma_{13}$	7957.4	32	8021.4	8184.6	8048.8	32	8112.7	8275.1
III	$\sigma_{23} = 0$	7962.5	32	8026.5	8189.7	8043.2	32	8107.2	8269.6
IV	$\sigma_{13} = 0$	7966.1	32	8030.1	8193.4	8060.7	32	8124.7	8287.1
V	$\sigma_{23} = \sigma_{13} = 0$	7971.7	31	8033.7	8191.8	8060.7	31	8122.7	8280.0

Note: Weights applied (see Table 1 in main text). Effective sample sizes used for calculating BIC is 1,214 and 1,181 for the low- and high-SES group, respectively.

Table S2: Results from ESRMs of Educational Attainment by Parental SES. Regression Coefficients (b) and Standard Errors (SE)

	Panel (A) Parental SES: Low						Panel (B) Parental SES: High					
	Educational attainment				Selection equation		Educational attainment				Selection equation	
Cultural capital input	Low		High				Low		High			
	b	SE	b	SE	b	SE	b	SE	b	SE	b	SE
Parental SES (cont.)	0.07	(0.19)	0.03	(0.47)	0.50***	(0.13)	0.30	(0.24)	0.43***	(0.12)	0.31**	(0.10)
Mother married (ref. not married)	0.09	(0.23)	0.16	(0.38)	0.28*	(0.14)	0.72**	(0.26)	0.21	(0.26)	0.35**	(0.12)
Number of siblings	-0.28**	(0.11)	0.05	(0.22)	-0.25***	(0.07)	0.10	(0.17)	0.04	(0.15)	-0.23	(0.08)
Race (ref. white)												
Black	0.63*	(0.26)	0.56	(0.41)	-0.51***	(0.14)	0.03	(0.27)	0.24	(0.29)	-0.69***	(0.14)
Hispanic	0.56*	(0.26)	0.15	(0.47)	-0.47**	(0.17)	-0.27	(0.27)	0.08	(0.40)	-0.60***	(0.17)
Male (ref. female)	-0.71***	(0.20)	-0.94**	(0.33)	-0.02	(0.10)	-0.68**	(0.24)	-0.78***	(0.20)	-0.08	(0.10)
Mother's birth year	-0.01	(0.05)	0.03	(0.08)	-0.05	(0.03)	-0.04	(0.06)	-0.11*	(0.06)	-0.03	(0.03)
Child's birth year	0.00	(0.04)	0.03	(0.07)	-0.02	(0.02)	-0.03	(0.05)	0.05	(0.05)	-0.04	(0.02)
Grandparents' years of schooling					0.05**	(0.02)					0.05**	(0.02)
Mother's expected years of schooling					0.07*	(0.03)					0.10**	(0.03)
ρ	-0.59		-0.59						-0.59			

Note: All explanatory variables centered on their overall weighted sample mean. Standard errors estimated using the bootstrap with 1,000 replications. Panel A contains estimates from Model II, whereas Panel B contains estimates from Model III (see Table S1).
* $p \le 0.05$, ** $p \le 0.01$, *** $p \le 0.001$ (two-tailed tests).